1974

Optimization of intraseasonal water allocation

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Optimization of intraseasonal water allocation

by

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A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1974

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CHAPTER I. INTRODUCTION

For many countries, regional and national economic development depends on increased agricultural production. Basically, there are two sources of higher production. First, additional resources may be employed while using existing production techniques. An example of using additional resources while technology is invariant would be the opening of additional lands to agriculture through new irrigation projects. Second, a more efficient production technique can be employed to achieve higher output from the same resource base or the same level of output can be realized with fewer resources. An example of the second source of increased productivity would be improvement of existing irrigation systems and practices to ensure more efficient use of available water.

Among natural resources, water is becoming one of the most limiting in agricultural production, particularly in arid and semiarid areas. In addition, the demand for water in industry, agriculture, and municipal use is steadily increasing throughout the world. Water supplies, however, are relatively fixed over time and must therefore be allocated among competing uses in such a manner that their contribution to the economic and social welfare of society is maximized.

Water for irrigation is one of the alternative uses of this resource. In many areas of the world, water supplies are inadequate for satisfying actual and potential needs of users in agriculture. This shortage can be as environmental conditions vary, in quantities available at certain times of the year, or total available for the irrigation season. Interyear shortages also often occur. Farmers are usually uncertain as to when a shortage will appear. This uncertainty requires careful planning so that water supplies are used efficiently.

Within a given farming situation, water may be used most profitably on one crop rather than partial irrigation on several crops. Further, it may be more profitable to irrigate less land so as to approach an adequate supply for fewer acres. As a guide to individual investment in irrigation systems, an appraisal is needed of the effect of incremental changes in water supply on profitable adjustments in farming systems and on farm incomes. Optimum utilization of additional water would enhance net farm incomes and contribute to the success of individual projects.

The economic evaluation of the potential use and development of water resources in agriculture requires estimates of technical and economic input-output relationships between water and crops. More specifically, water-use decisions are concerned with determining the optimal quantity and timing of application so as to maximize the user's objective.

The objective of this study is to present and apply various optimization models for determining the efficient use of water as applied to an individual crop. The information derived from this study is expected to assist farmers in allocating their limited water supply among competing crops in the most efficient way and in improving the timeliness of water applications. It is also expected that this information will facilitate better management of soil and water resources by policy makers.

Plan of Study

In the second chapter, an attempt will be made to develop an overall picture of the soil-plant-water system for readers not having a soil science background.

In the following three chapters, three distinct methodological approaches are structured for intraseasonal allocation of irrigation water. In Chapter 3, the model utilizes conventional production function analysis to derive the yield response function where water is the independent variable. Using this derived function and assumed price relationships, the optimal rates of irrigation water will be estimated. The criticism of this approach is that it fails to consider the timing of water application.

In Chapter 4, a soil moisture-plant growth simulation model will be used to estimate the crop growth over a growing season under alternative assumptions. Basically, simulation offers an alternative means of predicting crop response to irrigation strategies and weather conditions on the assumption that a soil moisture deficiency is likely to decrease potential yield.

In both the production function and simulation models, it is assumed that the quantity of water used in each growth stage is independent of the quantity used in all other stages and that the quantity of water is unlimited. In many areas, water is a limiting factor, and timing of irrigation is often more important in determining productivity than the quantity applied.

In Chapter 5, a dynamic programming model is used to determine the optimal distribution of a given quantity of irrigation water over the season. The effect of different soil moisture strategies in different stages of plant growth on total return will be calculated.

Finally, in Chapter 6, the results obtained under the three models are summarized and compared so as to make recommendations for improving water management and for planning future research in plant-water-soil relationships.

CHAPTER II. PRINCIPLES OF SOIL-WATER-PLANT RELATIONSHIPS

In this chapter, an attempt will be made to develop an overall picture of the soil-plant-water system for readers not having a soil science background. The coverage of topics is selective, and they are not necessarily discussed in equal detail.

Soil

To understand the role of water in crop production, soil properties and the relationship of soil to water must be examined. Soil is a storage device for plant nutrients and an important water reservoir that smooths out day-to-day fluctuations in water availability for plant use. In general, soils are made up of (1) mineral (inorganic) particles, (2) organic matter, (3) air, and (4) water. Two important physical properties of soil are texture and structure. These determine the soil capacity to hold water as well as its infiltration rate (the movement of water into and through the soil).

Soil texture

Soil texture refers to the relative proportion of particle size (sand, silt and clay) in a particular soil. Land use capability and methods of soil management are largely determined by soil texture. Generally, the best agricultural soils are those containing 10 to 20 percent clay, 5 to 10 percent organic matter, and the rest divided about equally between sand and silt. (Kohnke, 1968). Water and nitrogen availability to the plant is closely related to soil texture. As the texture becomes finer, availability of soil water and nitrogen to the plant usually increases. Table 1 is presented to illustrate the broad classes of soil texture.

Table 1. Soil-texture classification[a]

General terms		Basic soil texture classes
Sandy soils	Coarse-textured soils	Sands
		Loamy sands
Loamy soils	Moderately coarse textured soils	Sandy loam
		Fine sandy loam
	Medium-textured soils	Very fine sandy loam
		Loam
		Silt loam
		Silt
	Moderately fine textured soils	Clay loam
		Sandy clay loam
		Silty clay loam
Clay soils	Fine textured soils	Sandy clay
		Silty clay
		Clay

[a]Source: U.S. Dept. of Agric. (1950, p. 503).

Soil structure

Soil structure refers to the manner in which the soil particles are arranged in groups or aggregates. Soil structure does not supply any of the factors essential to plant growth, but it does influence practically all plant-growth variables. For example, soil structure affects the rate by which water enters and moves through the soil. It also affects aeration, root penetration, availability of plant nutrients and other factors. In other words, a good soil structure may be an indirect factor permitting plant-growth factors to function at optimum efficiency. To date, no universally accepted soil structure classification exists. This is partly due to the difficulty of making quantitative measurements of soil structure.

Water

Soil water intensely affects many physical and chemical reactions of the soil as well as contributes directly to plant growth. Production of any crop is dependent upon availability of water during the growing season.

Classes of soil water

Water is held in the soil as a film coating the soil particles and in the pore space between individual particles or aggregates. When water is added to a dry soil by either rain or irrigation, it fills the pore spaces and moves through the soil by both gravity and capillary forces. When all pores are completely filled, the soil is said to be saturated.

Soil water can be divided into three classes: (1) gravitational, (2) capillary, and (3) hygroscopic. Gravitational water moves freely downward under the influence of gravity. This rate of downward movement basically depends on the size of the pore spaces and the soil texture. Capillary water is that held in pore spaces by capillary forces. Capillary water moves more slowly than free water. It can move any direction but always to the area where soil moisture tension is high. Hygroscopic water is that held so tightly to individual soil particles that much of it is nonliquid and moves as a vapor. It is not available for plant use during the growing period.

After a thoroughly wetted soil drains several days, it reaches the upper limit of the available soil moisture range. At this point, soil moisture is at field capacity. The lower limit of the available moisture range is called the permanent wilting point. In general, the water storage capacity of soil is a function of its depth and physical composition. Moisture storage characteristics of a soil are very important at the time of irrigation, because they determine the amount of water that can be effectively applied at each irrigation and also influence the timing of irrigation. The general relationship between soil moisture characteristics and soil texture is presented in Figure 1.

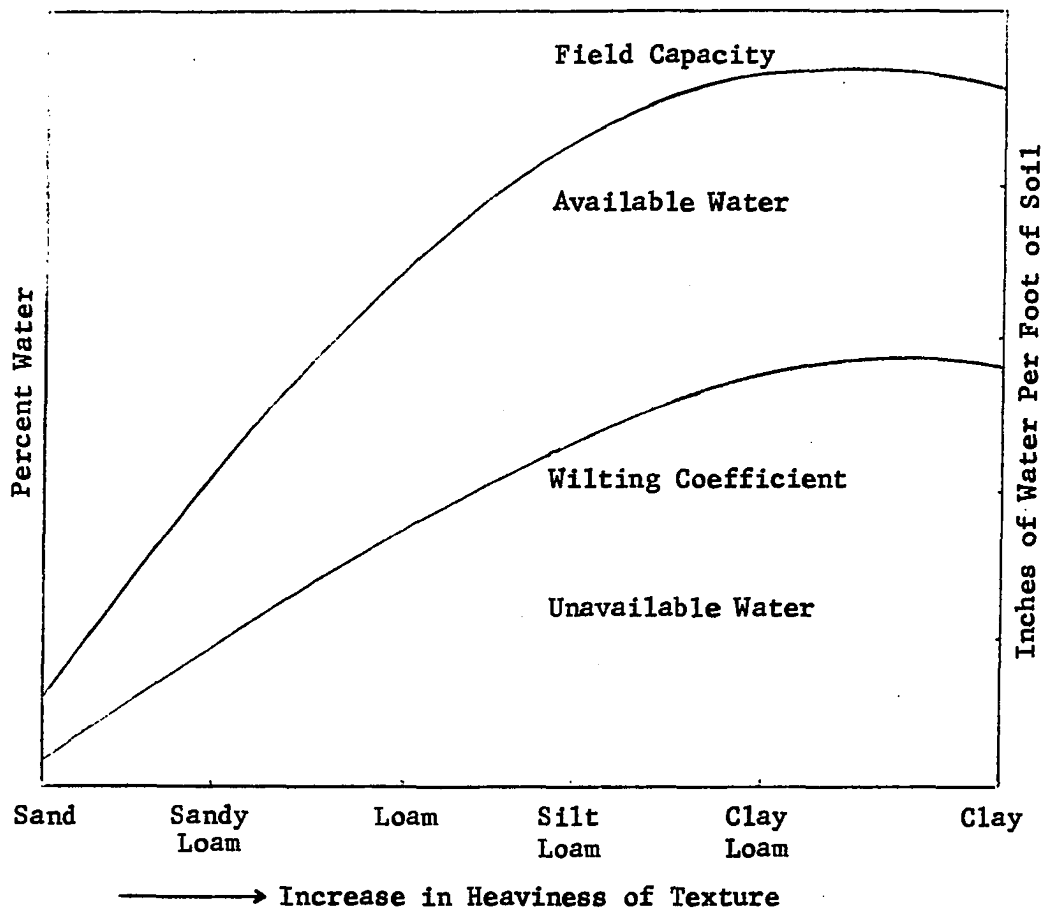

Figure 1. The general relationship between soil moisture characteristics and soil texture

<u>Movement of water in the soil</u>

Movement of irrigation water from the surface, into and through the soil is necessary for sustaining plant life and for removing surplus water. This water movement is dependent upon several factors including rates of infiltration (entering of water into soil) and percolation (downward movement through the soil). Both infiltration and percolation are important factors in determining the suitability of land for irrigation. The infiltration rate influences the rate at which water should be applied. Percolation removes excess water from the root zone and prevents a continued concentration of soluble salt that would otherwise accumulate in surface soil. On the other hand, percolation removes valuable plant nutrients beyond the root zone.

Soil moisture tension

Soil moisture tension is the force with which water is held in soil. Tension represents the energy required to remove water from the soil. Tension is usually expressed in equivalent atmospheres and an atmosphere is the average air pressure at sea level, i.e., 14.71 pounds per square inch.

Soil moisture tension depends on both the texture and structure of soils. Generally, fine-textured clays hold a considerable amount of moisture even at high tension, but sandy soils drain almost completely at low tension. Soil moisture tension measurements are useful in analyzing water movement and water usage by plants. Figure 2 shows moisture release curves for three soils of different texture. Tension values indicate the ease or difficulty with which moisture can be removed from the soil, and moisture percentages indicate the amount of water still in the soil. Field capacity and permanent wilting points are usually expressed in terms of soil moisture tension. Soil moisture tension levels of

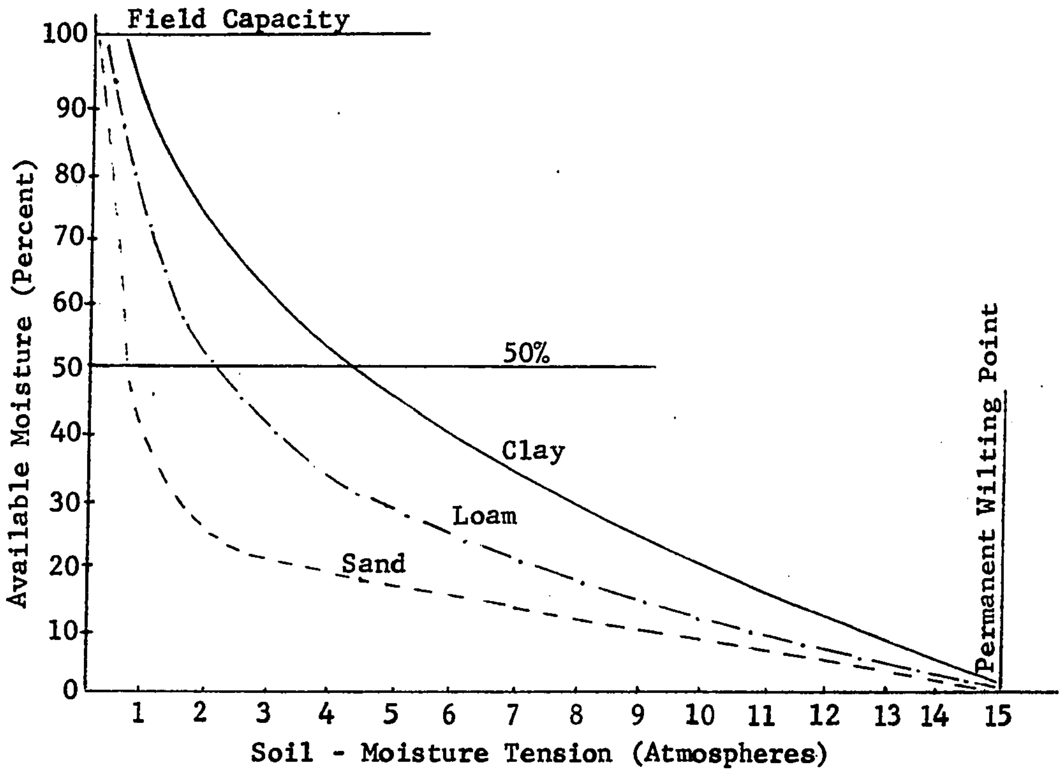

Figure 2. Moisture-retention relationship in the form of moisture-release curves for three soils of different texture and structure

1/3 and 15 atmospheres have been adopted by many soil scientists to designate the field capacity and the wilting point levels, respectively. As shown in Figure 2, tension at any moisture level is different for three soils. For example, at the 50 percent level, moisture tension for the sandy soil is 0.75 atmosphere; for loam, 2 atmospheres; and for the clay, 4.5 atmospheres.

Plant

In general, water makes up 90 percent or more of the weight of a green plant. Without enough water, physiological activity decreases, and gradually the plant ceases to grow, wilts and dies.

The functions of water in plant growth are numerous. Basically, water dissolves the nutrient elements thereby making them available to plants. In photosynthesis, water is an essential reagent as carbon dioxide. Water is also required to maintain sufficient plant turgidity for growth of cells and to support the leaves for capturing sunlight. The total quantity of water required for these essential functions is relatively small, usually less than five percent of the applied water. The rest of the water is subject to runoff, percolation and evapotranspiration. Evapotranspiration, often called consumptive use, represents the sum of evaporation of water from the soil and transpiration of water from the plant surfaces. Some of the factors that affect the rate of evaporation are the nature of evaporating surface, temperature, wind, and atmospheric pressure. Soil texture also affects evaporation. Evaporation is relatively lower in soils where water percolates freely. Factors that affect the rate of transpiration are moisture available in the soil, density of plant roots, atmospheric temperature, and soil fertility.

The total amount of water used in evaporation and transpiration by a crop varies over the growing season. Consumptive use is low at the beginning of the season. increases as plant foliage develops, reaches a peak during the fruiting period, and then rapidly declines as the plant reaches maturity.

<u>Rooting characteristics of the plant</u>

The amount of soil water available to a plant is determined partly by the depth and density of the root zone. Plants vary genetically in their rooting characteristics. Some have dense rooting systems and are able to use all the soil water within the root zone, while others have sparse roots preventing access to moisture supply at deeper soil layers. Figure 3 shows the effects of root density on the relation of growth to the depletion of available soil moisture. As shown in Figure 3 the sparser the roots, the greater the likelihood that growth will be retarded if irrigation is delayed.

Besides heredity, the root system is affected by such physical conditions of soil as bulk density, temperature, moisture and aeration. Chemical conditions, soil pH, fertility and salinity of soil can limit the growth of roots. Root penetration is also affected by soil layers. Root development is usually much greater in soils having layers of clay loam than in those of sandy textures. A layer of dry soil between a layer of moist soil acts as a barrier to extraction of water from the deeper layer.

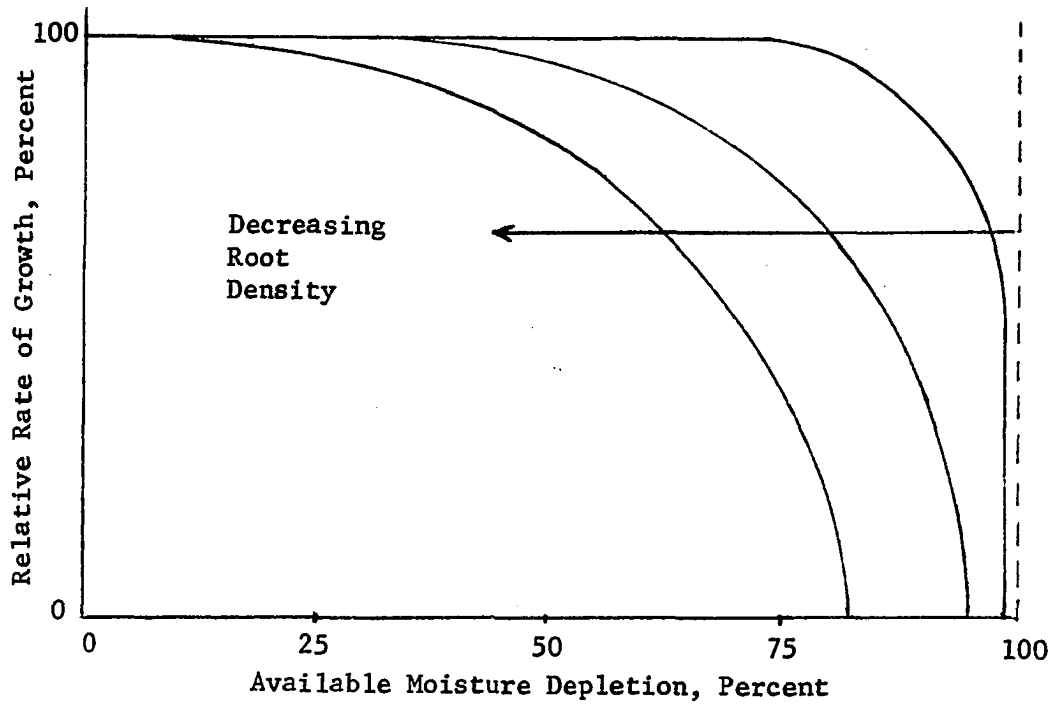

Figure 3. Effect of root density on the relation of growth to depletion of available soil moisture

<u>Internal water balance</u>

It is important to maintain a favorable water balance in plants if yields are to be maximized. This balance depends on the relative rates of water absorption and transpiration and is affected by the complex of soil, plant and climatic factors. The rate of absorption depends on the rate of transpiration, soil moisture availability and the rooting system of the plant. Aeration, temperature and moisture tension of the soil also affect the rate of absorption. As noted earlier, the transpiration rate is controlled by plant and atmospheric factors. Although these two processes are partly interdependent, transpiration is basically controlled by climatic factors and absorption basically by soil factors.

Internal water deficits occur when the rate of transpiration is greater than the rate of absorption. These deficits cause a lack of plant turgidity which, in turn, restricts the potential rate of growth. Rate of growth is generally a function of how hard the plant must work to absorb water from soil. The harder a plant must work to absorb water from soil, the slower it grows.

<u>Soil moisture requirement and plant growth</u>

The level of soil moisture is one of the most important factors affecting crop growth. There is no general agreement among irrigation agronomists with respect to the response of crops to various soil moisture regimes. One school of thought (Veihmeyer and Hendricksen, 1950) states that water is used with equal facility by plants between field capacity and permanent wilting point. In other words, it is likely unnecessary to reirrigate a given soil until the soil moisture has been reduced to the permanent wilting point. Another school (Hagan *et al.*, 1959) maintains that plant growth shows a differential response as soil moisture changes between field capacity and the permanent wilting point. In other words, the growth rate of plants, Y, is an inverse function of soil water stress, S, in the root zone. Summarizing, $Y = f(S^{-1})$. The latter school receives most recognition today. Both views are presented schematically in Figure 4.

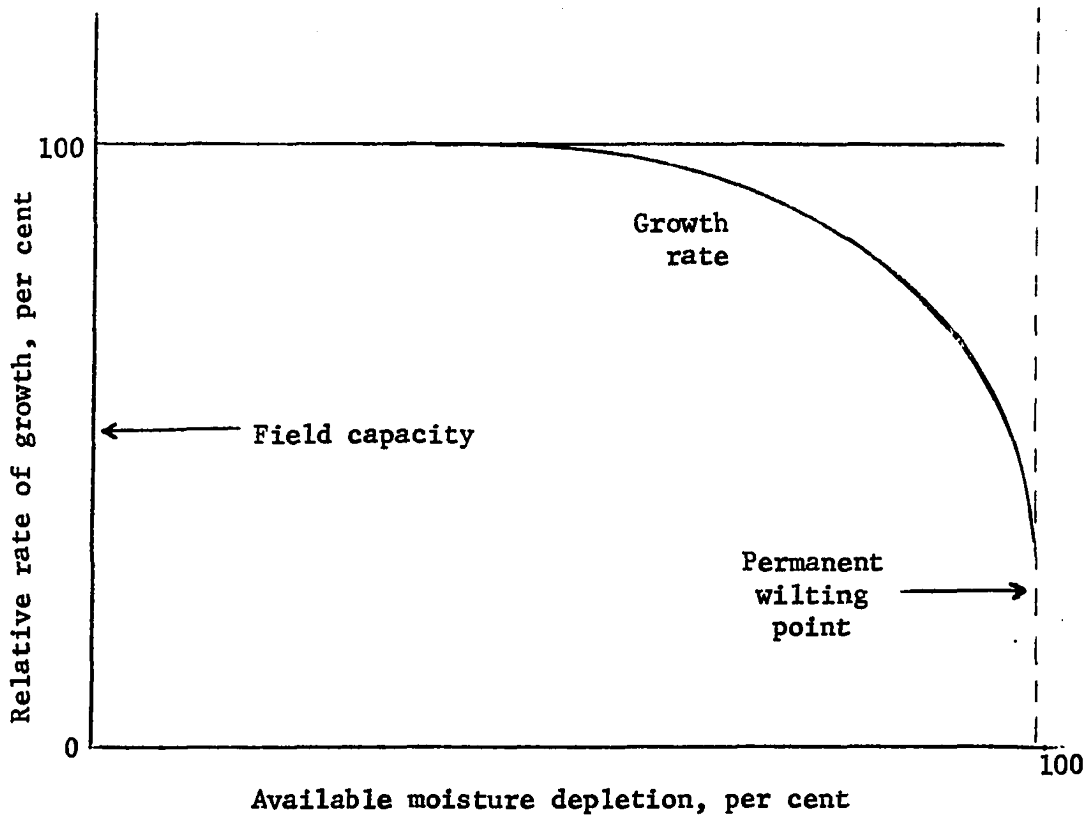

Figure 4. Relative growth at different levels of soil moisture depletion

Crop water requirements must be known when planning irrigation programs. Length of the growing season, stages of growth, soil factors, climatic conditions, and environmental factors must be taken into account. Most crops have critical periods during their growing season when a high level of moisture must be maintained for optimum yield. Some studies (Hagan et al., 1957) indicated that there are functional relationships between a plant's growth stages and the respective soil moisture levels, as in equation (1)

$$Y = f(X_i) \tag{1}$$

where Y is the total yield and X_i is the soil moisture level in stage i. In other words, given the soil characteristics, plant factors and climatic conditions, an adverse affect of soil moisture on the growth rate at any particular stage may affect subsequent growth and total yield.

CHAPTER III. THE PRODUCTION FUNCTION FOR CROPS USING IRRIGATION WATER

In the Western United States, as in many other semi-arid areas, water is one of the major factors in agricultural production. Farmers found that most soils in this area respond to water application by producing higher average yields and contribute toward stabilizing annual crop yields. Yet, the specific additions to yield resulting from given increments of water have not been adequately determined for many crops and soil combinations in various areas, nor has the economic level of application under various conditions.

From the information available it would appear that the amount of water used for irrigation in Western United States is not a very good indication of irrigation requirements in the area. Farmers with land located such that the source is from streamflow obtain water very cheaply and tend to over-irrigate when the water supply is adequate. With a shortage of water more efficient use normally results. As more water is used it becomes a larger portion of the production costs of crops and, therefore, of greater consequence to farmers. Thus, farmers are increasingly concerned with its efficient use, as water is becoming a limited factor in these areas.

Many factors, in addition to the quantity of water applied, affect crop response to irrigation water. Yields are affected by physical factors such as type and chemical properties of the soil, temperature, rainfall, previous crop rotations, insects and plant diseases, etc., and considerable variation occurs in many of these from one location to another. Thus, it is necessary to know the response function to water to make economically sound water recommendations for a crop that is applicable to an area in which it is grown. The primary concern of agricultural economists with this problem is in determining, for each crop, the level of irrigation water that maximizes profit.

The primary objective of this chapter is to empirically establish a production function for water (and nitrogen fertilizer) by abstracting from the detailed relationships involved in the growth process and concentrating on the general relation of corn grain to water (and nitrogen fertilizer) input. Following a brief review of literature on plant-water production function, a production function based on experimental data for irrigated corn is derived. Given this derived function and a particular price relationship, the rates of water yielding an optimum resource combination will be investigated.

Review of Literature

Since World War II, cooperation between agronomists and economists in designing and interpreting research to estimate the most profitable use-levels of various factors of production yielded very useful information. This information facilitated better management of soil and more efficient use of inputs by farmers and policy-makers.

Most of these studies relating to physical production functions are based on input-output relationships between fertilizers and crop yields. Heady and Dillon (1961) have given an excellent review of the development and application of production functions to input-output data.

Cooperative work on research into water-plant relationships has a shorter history. Beringer (1961) was one of the first to investigate these relationships. Agronomists and agricultural engineers independently developed numerous methods of estimating consumptive water use by plants. This is the quantity of water needed by the crop to maintain maximum growth. In early studies, the basic assumption was made that consumptive water use is constant for each crop. This assumption is based on the Viehmeyer-Hendrickson theory (1950). According to this theory, since water is equally available between the soil's permanent wilting point and field capacity[1], there is no need to irrigate so long as the soil moisture level stays above the permanent wilting point. As a result, the total quantity of water applied to any given crop is equivalent to the total amount of transpiration from that crop over its growing period. Further, given the expected climatic conditions, it is assumed that the water requirement of a crop is unique and constant. In contrast, Hagan et al. (1959) postulated that crops do not have a fixed level of consumptive water use.

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[1]Field capacity and permanent wilting point were defined on page 5.

Rather, plant growth shows a differential response as the soil moisture varies between field capacity and permanent wilting point.

Early water response studies considered the quantity of water applied plus rainfall as the variable input and crop yield as the output. A criticism of this "water quantity" approach was that it fails to determine the optimal allocation of water to a crop over an irrigation season. Furthermore, the results of such an experiment are only applicable to the particular soil type and climatic conditions existing when the experiment was carried out.

Beringer (1961) suggested relating plant growth to moisture tension and then indirectly to water quantity. Moisture tension is expressed as an index based on the aggregation of some measures of soil deficiency over the growing period. According to Beringer, introduction of this index has made possible the construction of water production functions which are more general and more independent of soil type. This approach, however, has also been criticized for not considering the importance of the time of occurrence of plant stress when soil moisture deficiencies arise.

Climatologists such as Wiser and Schilfgaarde (1964) have used "drought day" indices to investigate the yield response of crop to soil moisture deficiency. Baier and Robertson (1968) assumed that soil moisture should bear a closer relationship to plant growth and crop production than any single meteorological element. Further, they developed a soil moisture budget to estimate the yield response for wheat based on 39 plantings across Canada during a five season period. The yield estimates from the soil moisture model were compared with the direct use of climatological data in regression analysis. Multiple correlation analysis indicated that soil moisture was the better estimator. In addition to the studies by climatologists, several economists have related crop yield to soil moisture through weather variables and "drought days" indices. Engelstad and Doll (1961) investigated the weather differences on yield responses for corn to phosphorus applications. June and July rainfall together accounted for 75% of the variation in maximum yields of corn with increasing rates of applied phosphorus over a 12-year period at Grinville, Kentucky.

Two of the most important examples of "drought days" indices studied are those of Reutlinger and Seagraves (1962) and Smith and Parks (1967). Reutlinger and Seagraves eliminated the interyear effect through covariance analysis and estimated the functional relationship between soil moisture deficiency index and experimental tobacco yields. They combined the probability distribution of moisture deficiencies with the estimated production function to obtain expected yield and the variance of yield in relation to various irrigation applications.

Smith and Parks developed a simulation model which takes into account the effect of weather on crop response. Based on 35 years of data, the total number of drought days occurring during the growing season were estimated. Specifically, estimated yields and returns were determined on a probability basis for varying levels of nitrogen applied and product prices when drought was considered a random variable.

Several ventures of interdisciplinary cooperation among agronomists, engineers and economists incorporated the water input into the production function. One of the approaches was to estimate crop response to soil moisture stress (Voss and Pesek, 1967; Voss *et al.*, 1970; Corsi and Shaw, 1971). Voss and Pesek (1967) attempted to determine the effect of soil, management, and weather on corn yields grain to nitrogen, phosphorus, and potash treatments. Seasonal effect, as characterized by weather, was incorporated as stress days. The authors concluded that stress days in the five weeks following planting affected the yield response to applied nutrients.

Miller and Boersma (1966) incorporated water into the production function in two steps. First, they estimated the corn yield based upon soil moisture stress. Second, a regression was calculated between the amount of water available to the plant (irrigation plus rainfall) and minimum allowable stress.

The "water quantity" approach reflecting the quantity of water applied as a treatment in field experiments has been successfully used for estimating optimal water application. In most of these field experiments, it is assumed that the criticism of this approach, i.e., failing to consider the timing of water application, can be eliminated by making the distribution of water implicit in the experimental design.

Yaron (1971) analyzed more than 30 irrigation experiments and obtained statistically satisfactory results. He postulated that the curves fitted for a given crop in the same location but for different years tend to run parallel to each other. The difference in elevation among the curves may be explained in terms of differences in soil fertility and other factors in the particular year.

Stewart and Hagan (1969) have investigated alfalfa, wheat and grain sorghum production functions for several locations and concluded that convex (production) functions are most applicable for water allocation decision making.

Other studies (Kloster and Whittlesey, 1969; Kleinman, 1969; Stewart et al., 1971) have shown that the "water quantity" approach gives an adequate account of the relationship between applied water and yield responses for different crops. They all used polynomial functions and statistical analyses indicated significance responses to water applications.

The data that will be used in this study are a small part of a large-scale project between Iowa State University and the Bureau of Reclamation, U.S. Department of the Interior. Estimation of the response surfaces of corn, wheat, sugarbeets and cotton to irrigation water and fertilizer treatments in 6 western states has been completed and "generalized" production functions have been developed for each crop.

Production Function Analysis

A production function can be defined as a mathematical formulation expressing the technical relationship between the maximum amount of output that can be produced with each combination of specified factors of production given the existing technology. It may be expressed in the general implicit form for two inputs water (X_1) and nitrogen fertilizer (X_2) as:

$$Y = f(X_1, X_2) \tag{1}$$

X_1 and X_2 can take any pair of real values, and to any one pair of X_1 and X_2 values corresponds one value of Y which stands for output. In addition to water and nitrogen fertilizer, yield of a particular crop is a function of variables such as weather, seed variety, soil, and management. Each factor also contains a number of subfactors, each of which may be limiting or modifying. To include these additional factors, equation (2) is written as:

$$Y = f(X_1, X_2, X_3 \ . \ . \ . \ X_n) \tag{2}$$

Equation (2) implies that all input factors are variable. In reality, it is impossible to specify all inputs of production in any experiment. Only some factors are considered variable, the other factors either being held constant or regarded as having insignificant effects. This situation is indicated by rewriting equation (2) as:

$$Y = f(X_1, X_2 / X_3 \ldots X_n) \tag{3}$$

where the slash indicates factors X_3 to X_n are considered constant.

In empirical studies, a stochastic error term is added to account for the unspecified variables:

$$Y = f(X_1, X_2 / X_3 \ldots X_n) + e \tag{4}$$

The term e represents the deviation between predicted and actual yields. It is assumed that errors are randomly distributed with zero mean and a constant variance.

One of the problems in production function analysis is the determination of appropriate mathematical models that accurately approximate the observed input-output relationship. For plant-soil-water relationships, the choice of a proper functional model is still a methodological problem. To date, there is no single form that can be used to characterize response functions under all environmental conditions. Hence, the selection of an appropriate mathematical form depends on the particular phenomena under investigation. In general, models should satisfy certain theoretical properties. Typically and in the case of water-fertilizer experiments, knowledge of biological relationships requires that production functions allow a maximum yield and the possibility of a diminishing total yield. Further, the function should allow substitution and/or complementary relationships among inputs at different yield levels.

If previous knowledge cannot be used to specify an appropriate model, several types of functions are usually fitted to the experimental data and the "best" function selected by using various statistical criteria. One useful procedure is to examine the size of the "lack of fit" term derived in the analysis of variance. A nonsignificant lack of fit mean square indicates that the model is appropriate for the particular set of data being analyzed. Also, the magnitude of the coefficient of multiple determination (R^2) provides a basis for selection of a model. R^2 indicates the proportion of variance in the dependent variable accounted for by the multiple regression equation, i.e., the variables included in the model. Other factors equal, a higher R^2 is preferred. Other related statistics used as empirical criteria are the "t" values of independent variables. The magnitudes of the t's indicate independent variables which should be dropped because they are not statistically significant at acceptable probability levels. Nonsignificant variables may be retained when the have important theoretical or conceptual considerations.

Past research experiences should also be taken into consideration when selecting mathematical models to represent input-output relationships. The generally accepted types of function fall into three main categories: (1) exponential, (2) power function, and (3) polynomial. Each class of function is briefly discussed below.

Exponential function

The exponential function is also termed a Mitscherlich-Spillman type function. One form of this function is given in equation (5):

$$Y = A(1 - e^{-cX}) \tag{5}$$

where Y is output, A is maximum yield, e is the base of natural logarithm, c is a constant and X is the variable input. Some of the features embodied in an exponential function such as in equation (5) are that: (a) it does not allow a diminishing total yield; (b) it permits diminishing marginal productivities of the input applied; (c) inputs have independent effects on yield; and (d) the elasticity of production changes but the ratio of marginal products is constant over all ranges of input.

Power function (Cobb-Douglas)

The power function may be expressed in the form:

$$Y = aX^{b} \tag{6}$$

where equation (6) allows yield to increase at either an increasing, constant, or decreasing rate, but the response curve can be represented by only one of these and never by a combination. Yield in equation (6) does not have a maximum. Also, the elasticity of production is constant for each input use-level.

Both exponential and power functions are not appropriate for biological relationships where input use-levels are large enough to cause total product to decline.

Polynomial

Polynomial functions are more flexible relative to power and exponential functions. The general form is given in equation (7).

$$Y = X + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_{n-1} X^{n-1} \qquad (7)$$

where the X variable may be transformed to a square root, logarithmic reciprocal or some other form. Two of the most popular forms of the polynomial are the quadratic and square root functions expressed as equations (8) and (9) respectively.

$$Y = \alpha + \beta_1 X + \beta_2 X^2 \qquad (8)$$

$$Y = \alpha + \beta_1 \sqrt{X} + \beta_2 X \qquad (9)$$

Both functions define maximum yields, allow marginal products to diminish and permit both positive and negative marginal products. The basic difference between the two is that marginal product is diminishing at a decreasing rate in the square root function, but at a constant rate in the quadratic function. Further, the square root function increases very rapidly at low input use-levels, flattens out before reaching a maximum and then decreases very slowly.

No further attempt will be made to cover all forms of production functions and their properties here. The basic theory relating to production functions and procedures of estimation can be found in a number of sources (Ferguson, 1969; Dillon, 1968; Heady and Dillon, 1961).

Design of Experiment and Source of Data

One of the most ambitious sets of experiments ever undertaken to determine the basis for economic evaluation of irrigation development on individual farms was financed by the Bureau of Reclamation, U.S. Department of Interior, and conducted cooperatively with the Center for Agricultural and Rural Development and agronomists in the seven western states of the U.S.A. Correct economic and resource allocation decisions depend, in part, upon knowledge of production response to water. Therefore, an experimental design was chosen to estimate the production response in relation to irrigation water and its interaction with nitrogen fertilizer.

The specification and distribution of various treatment combinations was designed to facilitate estimation of coefficients for a production function of the second-order polynomial form. The experimental design used for most experiments was an incomplete block design involving factorial treatments with five levels of water and five levels of nitrogen. The block design is incomplete because all treatment combinations do not appear in each block of experiments. For example, the treatment combination of irrigation 1 and fertilizer 2 is not included in field experiments. Most experiments were compressed of two blocks with each block containing 22 plots. The combination of factors used in input space is given in Figure 5. Within a block, each treatment designated by an X was replicated twice and each designated as an 0 was replicated once. Consequently, there are 44 yield observations.

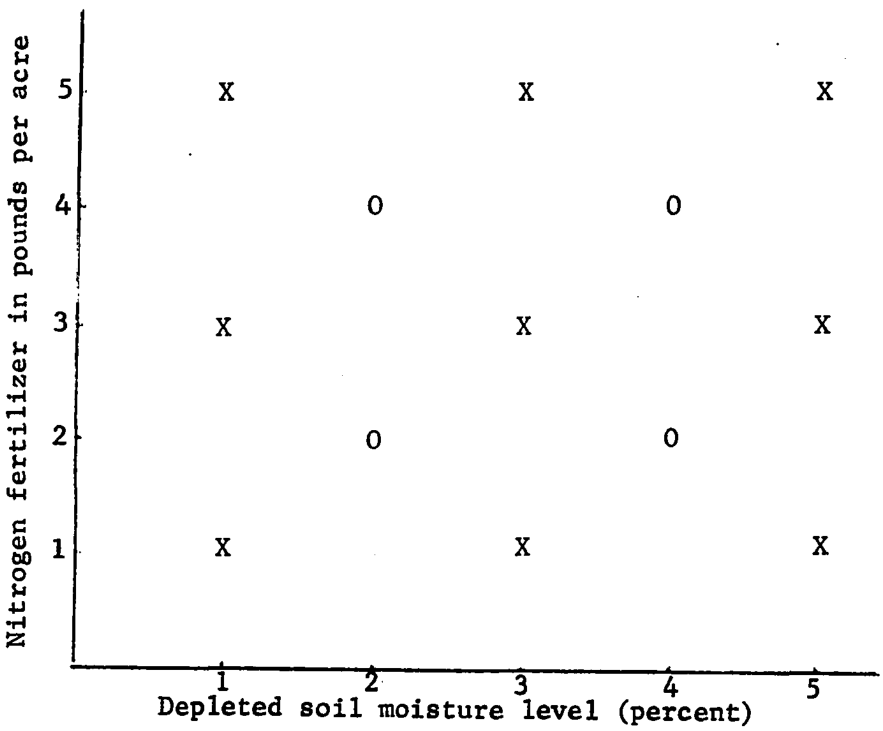

Figure 5. Experimental design showing combinations of factors which comprise the treatments

The data used in forming the production function and economic relationships for irrigated corn were obtained from 1971 nitrogen fertilizer and irrigation trials at the Colby Branch Station of Kansas State University. The plot size consisted of eight rows of corn with 30 inch spacing and 95 feet long. Prior to applying nitrogen fertilizer, residual nitrogen fertilizer was estimated for each plot. Phosphorus and pH determinations were also made for each plot, and phosphorus was applied to bring the soil level up to 54 pounds of available phosphorus per acre. Estimated soil moisture at the five foot level shows that all plots were approximately at field capacity during the first week of the experiment. The effective rainfall over the growing season is about 7.56 inches.

Based on knowledge of plant-soil relationships and previous empirical experiments, the agronomist of the Colby Station determined the appropriate fertilizer treatment levels.[1] Nitrogen fertilizer treatments ranged from 0 to 360 pounds per acre in increments of 90 pounds. The level and timing of irrigation treatments were designated to maintain available soil moisture at or below selected levels throughout the growing season. Actual irrigation treatments were based on applying water when the available soil moisture in the top two feet of soil was depleted to approximately 20%, 35%, 50%, 65%, and 80%. See Appendix A.

Each plot was bordered so that irrigation water could be

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[1]Mr. Evals Banbury, Colby Branch Station, Colby, Kansas.

measured into each basin and an even distribution of water attained. Irrometers were installed in each plot at a depth of 18 inches. These were used as an aid in determining when to sample the soil for soil moisture measurements for the 80% and 65% and to some degree 50% available soil moisture treatments. These were not reliable for the other lower soil moisture determinations.

Corn was planted on May 7, 1971, using Prarie Valley 40-5 hybrid at a seeding rate of approximately 26,000 seeds per acre. The plots were harvested on October 29, 1971. Corn grain yields were adjusted to 15.5 percent moisture and are given in Table 2.

Empirical Production Functions

Several alternative functional forms were considered as a basis for estimating the yield response of corn to nitrogen fertilizer and irrigation water. Least-square multiple regression analysis and the Gauss-Newton method were used for fitting the linear and non-linear regressions, respectively. Estimated functions and related tests as well as coefficients of multiple determination (R^2) are summarized in Table 3. Each of the six forms were fitted using the data for applied irrigation water (W) and applied nitrogen fertilizer (N).

Implicit characteristics of Cobb-Douglas and Mitscherlich functions are not logically appropriate for water-nitrogen fertilizer response functions. Because of their multiplicative nature if one input is zero, output must also be zero. Furthermore, none

Table 2. Corn grain yields in pounds per acre at 15.5 percent moisture corresponding to specified treatment combinations. Colby, Kansas, 1971.

Treatment		Block I		Block II	
Irrigation % ASM	Nitrogen (pounds/acre)	Replication 1	Replication 2	Replication 1	Replication 2
80[a]	0	2708	1560	1421	2969
50	0	3021	2992	1293	1276
20	0	1873	1711	956	1600
65	90	7214	-	6802	-
35	90	6518	-	5758	-
80	180	9266	9232	8321	8826
50	180	8205	8605	8605	8651
20	180	5984	6170	7144	6164
65	270	10154	-	7805	-
35	270	6790	-	7335	-
80	360	10409	9568	9667	8646
50	360	9371	8663	8200	8275
20	360	6309	6448	6593	6680

[a]The quantity of water applied was based on percent available soil moisture in each plot. For example, for those plots receiving treatment combination %ASM=80 and N=0, actual irrigation levels were 18.2 and 17.7 acre inches for the two plots in Block I and 14.4 and 21.5 for the two plots in Block II.

Table 3. Production functions for corn grain grown under varying levels of water and nitrogen fertilizer; "t" statistics, R^2, significance of "F" tests. Colby, Kansas, 1971.

Equation number[a]	Production functions	R^2	"F" tests
(1)	$Y = -1346.9214 + 433.0286W^{**} + 39.9385N^{**} - 10.9938W^{2**} - .0839N^{2**} + .3875WN^{**}$	.936	111.15
(2)	$Y = 2405.3837 + 826.4072W^{**} + 61.8810N^{**} - 126.5854W^{1.5**} - 2.7662N^{1.5**} + .423WN^{**}$	.944	128.11
(3)	$Y = -10725.8440 - 819.6973W^{**} - 19.6239N^{**} + 6585.8594W^{.5**} + 363.0603N^{.5**} + 84.9409W^{.5}N^{.5**}$	.955	161.29
(4)	$Y = 982.6289 + 171.5141W^{**} + 14.9631N^{**}$	.764	61.51
(5)	$Y = 730W^{.3783**}N^{.2469**}$	.919	234.03
(6)	$Y = 12189.547\,(1 - e^{-.0296W*})(1 - e^{-.0121N*})$	.721	94.3

[a](1) Quadratic function; (2) 1.5 power function; (3) Square root function; (4) Linear function; (5) Cobb-Douglas function; (6) Mitscherlich function.

**Significant at 5% level.

*Significant at 10% level.

of the functions exhibits a finite maximum, i.e., marginal physical products are always positive. Correspondingly, their isoclines emanate from the origin and do not converge.

Heady and Dillon (1961) postulated that polynomial forms were satisfactory for describing agricultural yield responses. Generally, the polynomials are completely flexible functions in that all degrees of input substitution are allowed to exist. Statistical tests of significance for the derived coefficient are easily applied.

The statistical tests provide little evidence for the preference among the three polynomial forms, i.e., quadratic, square-root, three-halves. The quadratic function given in equation (11) is chosen for the in-depth analysis of the corn data.

$$\begin{aligned} Y = -1346.9214 + 433.0286W + 39.9385N \\ - 10.9938W^2 - .0839N^2 + .3875\ WN \end{aligned} \tag{11}$$

In equation (11), Y is the predicted corn grain yield expressed as pounds per acre, and W and N are the water applied in acre inches and nitrogen fertilizer applied in pounds per acre, respectively. The magnitude of R^2 shows that 93.6% of the variation in yield is explained by using the applied water and nitrogen fertilizer variables. Except intercept, all of the coefficients are significant at the .05 probability level. The negative signs of the estimated coefficients for W^2 and N^2 denote diminishing marginal returns for water and nitrogen fertilizer. The last term in equation (11) represents the interaction between fertilizer and water, and its positive sign indicates a complementary relationship in the production of corn.

To predict the optimum rate of nitrogen fertilizer, available soil nitrogen prior to planting is taken into consideration. It is assumed that available soil nitrogen fertilizer and applied nitrogen fertilizer contribute identically to plant response and are, therefore, additive. The response to total fertilizer and applied water is described by a second degree polynomial, equation (12).

$$Y = -2297.9782 + 354.7406W + 42.9540N - 9.5524W^2 - 0.0761N^2 + 0.4211WN \tag{12}$$

Again, Y is the predicted corn yield, W is the amount of water applied in acre inches and N is the nitrogen fertilizer applied plus residual available soil nitrogen fertilizer. The derived coefficient for the intercept is statistically significant at the .10, and the rest of the coefficients are significant at the .05 probability level. The value of the coefficient of determination, R^2, was not changed very much, and the model is about 93% deterministic. The predicted maximum yield from equation (12) is 10,014 pounds per acre of corn when 26.4 inches of water and 355 pounds of nitrogen fertilizer are applied. These results are almost identical to the maximum yield predicted from equation (11) which is 9,945 pounds per acre of corn based on use-levels of 24.9 inches of water and 295 pounds of nitrogen fertilizer. The major difference between the two yield-maximizing use-levels of fertilizer can be explained by the addition of residual soil nitrogen fertilizer to the applied nitrogen fertilizer in equation (12).

To investigate the corn response where the water variable is defined in terms of the quantity of water used, the following equation was fitted to the data:

$$Y = -10763.3830 + 887.5356W + 22.1226N - .0540W^2 - .0796N^2 + .6907WN \tag{13}$$

where Y is estimated yield of corn measured in pounds per acre, N is the applied nitrogen fertilizer in pounds per acre and W is defined as water used by the plant. That is, water use-levels are estimated as available soil moisture at planting plus rainfall during the growing season plus the quantity of water applied minus available soil moisture at harvesting. All of coefficients in equation (13) are significant at the .05 probability level and the value of R^2 is .937. Comparing equations (11) and (13) in terms of predicted maximum yields, the estimated maximum yield with equation (13) is 10,033 pounds per acre of corn when 39.2 inches of water and 306 pounds of nitrogen fertilizer are used. The implication of equations (11) and (13) is that the difference between the use-levels of water applied and consumptive water use which maximize yields is about 14 inches per acre. This difference is approximately equal to available soil moisture at planting plus rainfall minus available soil moisture at harvesting.

A comparison of equations (11), (12), and (13) reveals that not much is gained by including residual nitrogen and water other than applied irrigation water. Thus, inferences drawn throughout the remainder of this chapter are based upon relationships implicit in equation (11).

Prediction of yields

Assuming the influence of other inputs such as labor and machinery constant, corn yields can be predicted for specific water and fertilizer rates by using equation (11). Predicted yields over the experimental range of water and nitrogen fertilizer are given in Table 4. When both inputs are increased simultaneously, corn yield increases by more than the sum of the increase in yield attained by increasing each input individually. In Table 4, for example, with fertilizer at 100 pounds per acre and applied water at 10 acre inches, increasing the water rate to 15 inches increases yield by 984.67 pounds. But holding water at 10 acre inches and increasing fertilizer to 150 pounds per acre increases yield by 1,141.92 pounds. Summing these two quantities implies the yield increase would be 2126.59 pounds. If water and fertilizer are simultaneously increased to 15 acre inches and 150 pounds, respectively, yield increases by 2223.37 pounds. These results show the effect of a positive interaction between water and fertilizer on yield.

Table 4. Predicted corn grain yields[a] at specified levels of nitrogen fertilizer and water, Colby, Kansas, 1971

Nitrogen fertilizer applied (lb./acre)	Water applied (acre/inches) 0.0	5.0	10.0	15.0	20.0	25.0	30.0
0	1346.91	543.39	1884.00	2674.92	2916.15	2607.70	1749.55
50	440.26	2427.44	3864.92	4752.72	5090.83	4879.25	4117.97
100	1807.94	3891.99	5426.35	6411.02	6846.00	6731.30	6066.90
150	2756.11	4937.04	6568.27	7649.82	8181.68	8163.85	7596.32
200	3284.79	5562.59	7290.70	8469.12	9097.85	9176.90	8706.25
250	3393.96	5768.64	7593.62	8868.92	9594.53	9770.45	9396.67
300	3083.64	5555.19	7477.05	8849.22	9671.70	9944.50	9667.60

[a]Yields are in pounds per acre at 15.5 percent moisture.

Each row in Table 4 gives predicted yields for a single variable response function Y=f(W) with nitrogen fertilizer fixed at the level specified for that row. Moving across the rows in the Table 4, corn yields increase, reach a maximum, and then decrease as the water rate increases. Likewise, each column gives a Y=f(N) curve, with water fixed at the level specified for that column.

Figure 6 geometrically illustrates the production surface predicted by equation (11) for varying rates of water and fertilizer. The slope of the surface indicates the response to both water and fertilizer. The slope is greater along the nitrogen fertilizer axis than along the water axis; the steeper slopes correspond to the greater response to nitrogen fertilizer as compared to water in Table 4.

The production surface illustrates high marginal products for the first 20 inches of water. Beyond 20 inches, however, the predicted response of corn yields to water diminishes. The highest marginal response for nitrogen fertilizer comes with the first 200 pounds of fertilizer. After 200 pounds of fertilizer, the predicted response of corn yields to nitrogen fertilizer flattens out and diminishes slightly at 300 pounds of fertilizer. The surface also slopes upward from the center-corner above the production surface because of the water by fertilizer interaction. A slice through the surface parallel to the water axis in Figure 6 would represent response of corn to water at a fixed level of nitrogen fertilizer.

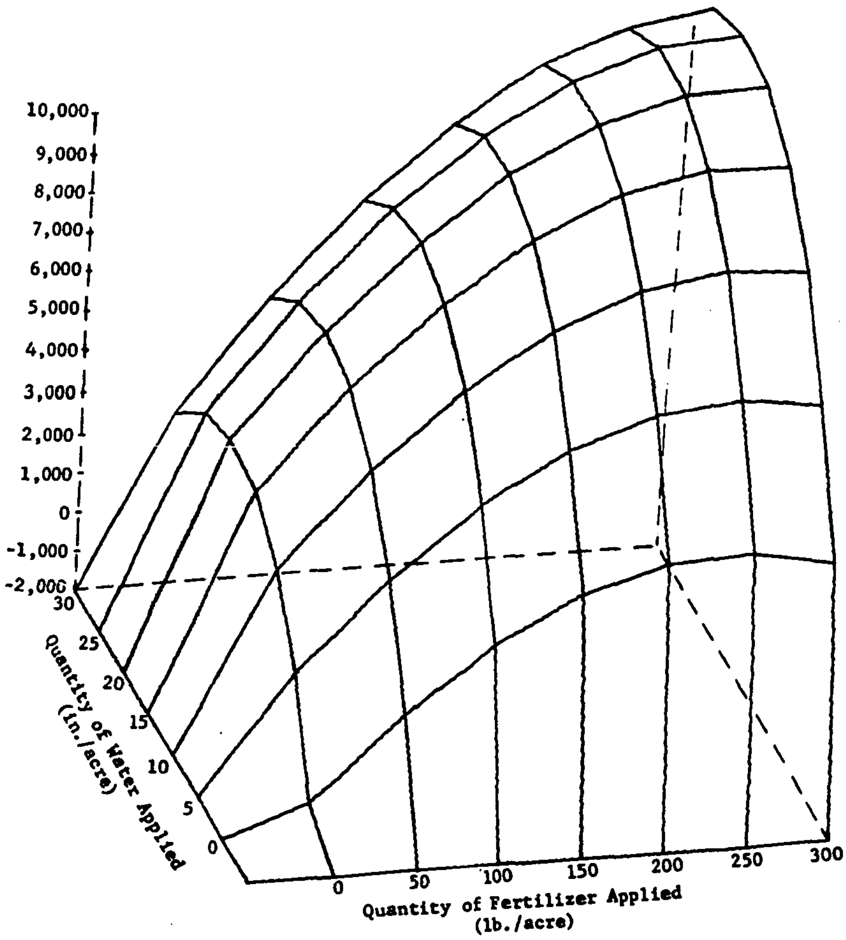

Figure 6. Production surface for corn predicted by quadratic function (11), Colby, Kansas, 1971

Marginal physical products

The marginal physical product of an input is the addition to the total product obtained by using one additional unit of input. The marginal physical products of water and nitrogen are obtained directly as the first derivatives of the response function given in equation (11). These are given in equations (14) and (15), respectively.

$$\frac{\partial Y}{\partial W} = 433.0285 - 21.9876W + .3875N \tag{14}$$

$$\frac{\partial Y}{\partial N} = 39.9385 + .3875W - .1678N \tag{15}$$

Tables 5 and 6 provide the marginal physical products of water and fertilizer for corn obtained by substituting the specified input values into equations (14) and (15), respectively. When nitrogen fertilizer is held constant at zero, the marginal physical product of water becomes negative when its level is above 19.69 acre inches. Similarly, when water is held constant at the zero level[1], the marginal physical product of nitrogen fertilizer becomes negative when its level is above 238.01 pounds per acre. The marginal physical product of water increases as the level of fertilizer is increased and vice versa. See Tables 5 and 6.

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[1]At zero level of irrigation water applied, the crop will still have some moisture through available soil moisture and rainfall.

Table 5. Marginal physical product of water at different levels of nitrogen fertilizer in the production of corn at Colby, Kansas, 1971

Nitrogen fertilizer applied (lb./acre)	Water applied (acre/inches)						
	0.0	5.0	10.0	15.0	20.0	25.0	30.0
0	433.03	323.09	213.15	103.22	-6.72	-116.66	-226.60
50	452.40	342.47	232.53	122.59	12.65	-97.29	-207.22
100	471.78	361.84	251.90	141.97	32.03	-77.91	-187.85
150	491.15	381.22	271.28	161.34	51.40	-58.54	-168.47
200	510.53	400.59	290.65	180.72	70.78	-39.16	-149.10
250	529.90	419.97	310.03	200.09	90.15	-19.79	-129.72
300	549.28	439.34	329.40	219.47	109.53	-0.41	-110.35

Table 6. Marginal physical product of nitrogen fertilizer at different levels of water in the production of corn at Colby, Kansas, 1971

Nitrogen fertilizer applied (lb./acre)	Water applied (acre/inches)						
	0.0	5.0	10.0	15.0	20.0	25.0	30.0
0	39.94	41.88	43.81	45.75	47.69	49.63	51.56
50	31.55	33.49	35.42	37.36	39.30	41.24	43.17
100	23.16	25.10	27.03	28.97	30.91	32.85	34.78
150	14.77	16.71	18.64	20.58	22.52	24.46	26.39
200	6.38	8.32	10.25	12.19	14.13	16.07	18.00
250	-2.01	-0.07	1.86	3.80	5.74	7.68	9.61
300	-10.40	-8.46	-6.53	-4.59	-2.65	-0.71	1.22

Setting the marginal product equations (14) and (15) equal to zero and solving simultaneously for water and fertilizer, the maximum yield and the corresponding input quantities can be derived. The derived maximum yield is 9945.46 pounds per acre of corn obtained with 24.9 inches of water and 295.496 pounds of nitrogen fertilizer. However, an application of 19.69 acre inches of water results in a maximum yield of 2917.16 pounds of corn when nitrogen fertilizer is held constant at zero. Similarly, when water is held constant at zero level, a maximum yield of 3406.00 pounds per acre of corn grain can be obtained with 238 pounds per acre of fertilizer.

Yield isoquant

In corn production, it is possible within limits to substitute water for fertilizer and vice versa. To consider these substitution possibilities, isoquant contours can be calculated if the estimated production functions meet the sufficient conditions of a production function that $\frac{\partial^2 Y}{\partial W^2} < 0$. The isoquant function for water $W = f(N,\bar{Y})$ or for fertilizer $N = f(W,\bar{Y})$ embodies all combinations of water and nitrogen fertilizer that produce a specific level of output, $\bar{Y}$. The isoquant equations were derived by alternatively solving the production function in equation (11) for each individual input as a function of the other input and yield. Equation (16) is the isoquant function expressing water as a function of nitrogen fertilizer and yield.

$$W = \frac{(433.0285 + .3874)}{2(10.9938)}$$

$$\pm\frac{[(433.0285 + .3874N)^2 - 4(10.9938)(Y + .0839N^2 - 39.9385N + 1346.9214)]^{.5}}{2(10.9938)} \tag{16}$$

The nitrogen fertilizer and water treatment levels derived for specified yields substituted into equation (16) are tabulated in Table 7 and graphed in Figure 7. The production function exhibits diminishing return to each of the inputs. This principle is also shown by the spacing of the isoquants in Figure 7. The distance between the isoquants increases as the yield rises, thereby indicating that proportionately greater quantities of inputs are required to obtain high yields as compared to low yields.

The convexity of the isoquants indicates that water and fertilizer are less than perfect substitutes. In fact, decreasing marginal rates of substitution exist. The marginal rates of substitution of nitrogen fertilizer for water would be defined as the amount of fertilizer that could be replaced by an acre-inch of water. Movement along any single isoquant shows that it takes increasing quantities of nitrogen fertilizer to substitute for a unit of water. The marginal rate of substitution of nitrogen fertilizer for water, MRS_{NW}, can be expressed as a negative inverse ratio of their marginal products. The MRS_{NW} is given in equation (17).

Table 7. Values of selected isoquants showing combinations of water and nitrogen fertilizer required to produce specified yield levels and corresponding marginal rates of substitution of water for nitrogen fertilizer. Colby, Kansas, 1971

Corn grain yields (lb./acre)	Nitrogen fertilizer applied (lb./acre)	Water applied (in./acre)	Marginal rates of substitution ($\partial W/\partial N$)
5000	50	17.66	-1.67
.	100	8.42	-10.09
.	150	5.17	-22.51
.	200	3.66	-55.15
.	250	3.26	+[a]
.			
6000	50	-[b]	+
.	100	12.57	-6.51
.	150	8.07	-17.53
.	200	6.14	-42.88
.	250	5.58	-270.55
.			
7000	50	-	+
.	100	-	+
.	150	11.72	-12.09
.	200	9.05	-31.52
.	250	8.22	-297.67
.			
8000	50	-	+
.	100	-	+
.	150	17.68	-4.94
.	200	12.74	-20.36
.	250	11.41	-115.83
.			
9000	50	-	+
.	100	-	+
.	150	-	+
.	200	18.89	-6.95
.	250	15.73	-45.07

[a]"+" - Substitution of water for nitrogen fertilizer is outside the economic region.

[b]"-" - Water level is negative.

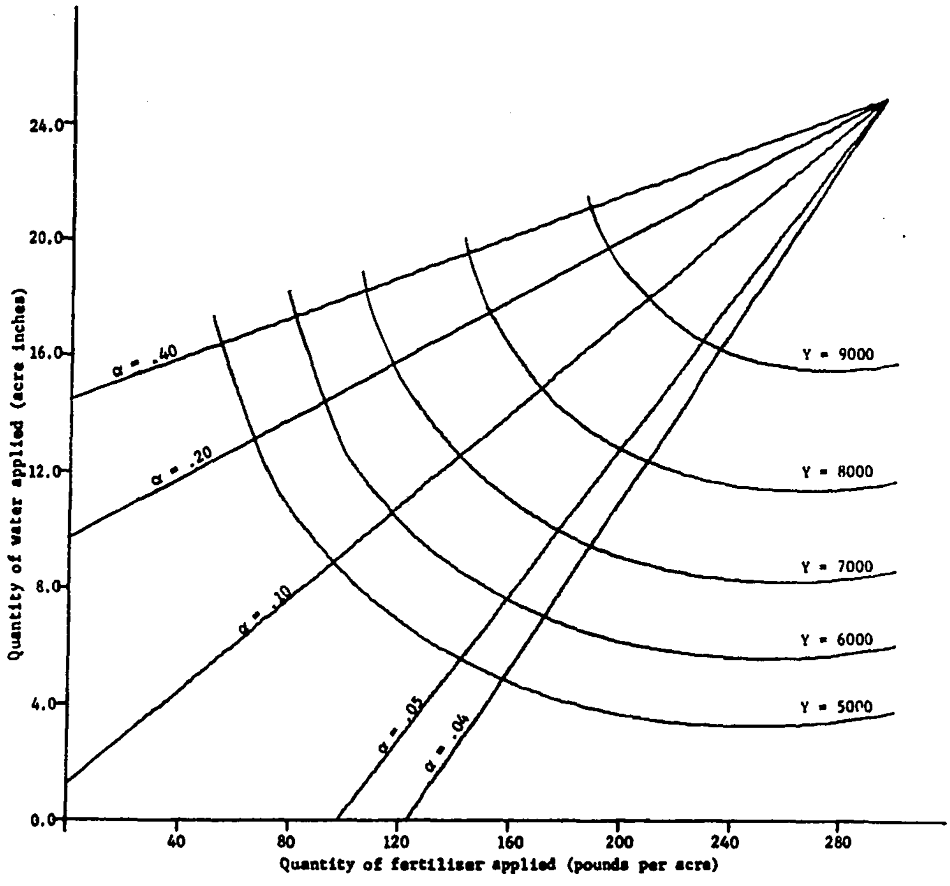

Figure 7. Yield isoquants and isoclines for corn predicted by quadratic function (11), Colby, Kansas, 1971

$$MRS_{NW} = - \frac{MPP_W}{MPP_N} = - \frac{(433.0285 - 21.9876W + .3874N)}{39.9385 + .3874W - .1678N} \tag{17}$$

Combinations of W and N required to produce a given level per acre of corn and corresponding marginal rates of substitution of water for nitrogen are given in Table 7. For example, with a yield isoquant of 5000 using 17.66 inches of water and 50 pounds of nitrogen fertilizer, one additional unit of water would replace only 1.67 units of nitrogen fertilizer in production. However, as nitrogen fertilizer is increased to 100 pounds and water is reduced to 8.42 inches, one additional unit of water would replace 10.09 pounds of fertilizer. The implication of this is that to maintain the same yield it takes larger quantities of fertilizer to substitute for a unit of water.

Given the marginal rates of substitution between inputs and given the input prices, it is possible to estimate the least-cost combination of inputs required to produce a given output. The point at which the slope of the price line, P_N/P_W, is equal to the slope of the isoquant, MRS_{NW}, defines the least-cost combination of producing the yield represented by that isoquant. P_W is the price of an acre inch of water and P_N is the price of a pound of nitrogen fertilizer.

Isocline

Based upon specified price relationships, isoclines connect points of least-cost combinations of water and nitrogen fertilizer for producing the yields represented by the family of isoquants. Equations of isoclines in Figure 6 were found by setting the ratio of the marginal physical products equal to the water-nitrogen fertilizer price ratio. Letting α equal the P_N/P_W ratio, the isocline equation is given in equation (18)

$$W = \frac{433.0285\alpha - 39.9385}{.3874 + 2(10.9938)\alpha} + \left[\frac{.3874\alpha + 2(.0839)}{.3874 + 2(10.9938)\alpha} \right] N \qquad (18)$$

Isoclines for several selected price ratios (α) are tabulated in Table 8 and graphed in Figure 7, respectively. As seen in Figure 7, isoclines are straight lines and converge at the water-nitrogen fertilizer combination that gives the maximum yield predicted earlier. Consider isoquant Y = 5000 in Figure 7. As water becomes expensive relative to fertilizer, the slopes of the points at which successive isoclines intersect the isoquant decreases. This indicates that the amount of water in the resource mix should be reduced and fertilizer increased, moving left to right on the isoquant. If nitrogen fertilizer becomes expensive relative to water, movement on the isoquant would be from right to left.

Table 8. Values of selected isoclines showing the optimum water and fertilizer combination to produce corn grain for varying water prices and a fertilizer price of $0.80 per pound, Colby, Kansas, 1971

Price of water ($/inches)	Price ratio ($\alpha=P_N/P_W=.08/P_W$)	Nitrogen fertilizer (lb./acre)	Water (inches)
.20	.40	0	14.51
.	.	50	16.27
.	.	100	18.03
.	.	150	19.79
.	.	200	21.55
.	.	250	23.30
.	.	300	25.06
.40	.20	0	9.75
.	.	50	12.32
.	.	100	14.88
.	.	150	17.45
.	.	200	20.01
.	.	250	22.58
.	.	300	25.14
.80	.10	0	1.30
.	.	50	5.30
.	.	100	9.30
.	.	150	13.29
.	.	200	17.29
.	.	250	21.29
.	.	300	25.28
1.40	.05	0	+[a]
.	.	50	+
.	.	100	.30
.	.	150	6.60
.	.	200	12.91
.	.	250	19.21
.	.	300	25.51
2.00	.04	0	+
.	.	50	+
.	.	100	+
.	.	150	3.87
.	.	200	11.12
.	.	250	18.54
.	.	300	25.60

[a]"+" - Indeterminate.

Marginal rates of substitution among water, nitrogen fertilizer and land

One of the economic aspects of crop response to irrigation water is closely related to the government acreage control program. This is the possibility of substituting water for land within a given crop production scheme. Specifically, the possibilities of producing the same yield from a larger area of land with less water or, more likely, the same or greater yield with less land and more water are considered.

When irrigation water is applied in semi-arid areas, per acre yields are usually increased. When acres of land are taken out of production of one crop such as corn, they become available for some other crop or for some other use such as recreation. Knowledge of these marginal rates are very useful in developing countries where the food supply lags and land is a limiting factor on production. If there is a significant potential for substituting water for land, then it may be possible to meet the growing need for land without incurring serious shortages.

To estimate the marginal rates of substitution for land, the water-fertilizer production function must be transformed into land-water-fertilizer production function where land is considered variable in quantity. The method of transformation is given by Heady (1963). The transformed production function is given in equation (19).

$$Z = -1346.921A + 433.028W + 39.938N - 10.933\ W^2/A - .0839N^2/A + .3874\ WM/A \tag{19}$$

where Z is pounds of corn grain and A is land measured in acres. From this function, the marginal rates of substitution among water, land and fertilizer can be derived by taking the total differential of (19) from which the marginal physical products of W, N and land can be derived. Taking the inverse ratio of two marginal products while keeping the third input fixed, the corresponding equations are given in equations (20), (21), and (22).

$$\left.\frac{dA}{dW}\right|_{dN=0} = \frac{-433.028 + 21.986\ W/A - .3874\ N/A}{-1346.921 - 10.993\ \frac{W^2}{A^2} - .0839\ \frac{N^2}{A^2} + .3874\ \frac{WN}{A^2}} \tag{20}$$

$$\left.\frac{dA}{dN}\right|_{dW=0} = \frac{-39.938 + .1678\ N/A - .3874\ W/A}{-1346.921 - 10.993\ \frac{W^2}{A^2} - .0839\ \frac{N^2}{A^2} + .3874\ \frac{WN}{A^2}} \tag{21}$$

$$\left.\frac{dN}{dW}\right|_{dA=0} = \frac{-433.028 + 21.986\ W/A - .3874\ N/A}{39.938 - .1678\ N/A + .3874\ W/A} \tag{22}$$

These marginal rates of substitution are "gross" because machinery and other capital items as well as labor associated with water and nitrogen applications and per-acre yield increases are not included in the study. For example, a given quantity of water that replaces certain acres in maintaining a fixed level of production, would also involve less machinery, less fertilizer, etc., to be used in a smaller acreage.

Economic optima

Deriving economically optimum use-levels for water and fertilizer in individual crop production under certainty and unlimited capital focus on selection of (a) the levels of water and fertilizer which will maximize the per-acre profits and (b) the combination of water and fertilizer levels which minimize the cost of a given output. These conditions can be attained by equating the partial derivatives of the production function with respect to water and fertilizer to their respective prices divided by the price of corn, as in equations (23) and (24) respectively, and solving these equations simultaneously. In equations (8) and (9), the prices of water, nitrogen fertilizer and corn have been assumed to be \$0.60/acre inch, \$0.08/pound of N, and \$0.024/pound of corn, respectively.

$$\frac{\partial Y}{\partial W} = 433.0285 - 21.9876W + .3874N = \frac{P_W}{P_Y} = \frac{.60}{.024} \tag{23}$$

$$\frac{\partial Y}{\partial N} = 39.9385 + .3874W - .1678N = \frac{P_N}{P_Y} = \frac{.08}{.024} \tag{24}$$

The derived profit-maximizing use-levels of water and fertilizer are 23.35 inches of water and 272.07 pounds of fertilizer.

The corresponding profit-maximizing yield is 9886.95 pounds of corn per acre. As would be expected, optimum water use-levels very as corn and nitrogen fertilizer prices vary. When water costs $0.80 per acre inch, the predicted optimum use-levels of water are lower, ranging from 18.4 to 22.7 inches per acre depending on nitrogen fertilizer and corn prices. Fixing the price of corn at $0.024/ pound and the price of water at $0.60/acre inch, and increasing fertilizer price to $0.16/pound will cause the optimum nitrogen fertilizer use-level to drop to 251 pounds.

Demand for water

Knowledge about the level and the elasticity of demand for water is useful in a number of different ways in decision-making. Water demand functions provide guides for allocation of present and future water supplies among various uses in an economically efficient manner. They help water agencies to determine the optimal pricing and valuation policies of available water at the farm or regional level. Availability of reliable water demand functions are also important from the policy standpoint of public investment in alternative water resources projects.

Demand functions for water can be derived from technical production functions. These demand functions are termed normative since they indicate what demand function would be if farmers maximize profits under the conditions where capital is unlimited and there is no uncertainty. Various aspects of the static-normative nature of demand and supply functions are discussed in Heady and Tweeten (1963).

Given the production function a static, short-run demand equation can be derived as a function of the amount of nitrogen fertilizer applied, the price of water and the price of corn. The term 'static' is used because it is supposed that the corn and input prices and the production function are known with certainty. The term short run means that the level of one input is fixed, e.g., N, and substitution of one factor for another is not permissible.

The short-run static demand function for water is derived by equating marginal product equation (14) to the water-corn price ratio and solving for the water variable, as in equation (25).

$$W = \frac{[433.0285 + .3874N - (P_W/P_C)]}{21.9876} \tag{25}$$

By specifying values for nitrogen fertilizer (N) and the corn price (P_Y), a family of short-run static demand equations is generated and presented in Table 9, and illustrated in Figure 8. The price of corn is fixed at $0.024 per pound. As shown in Figure 8, the demand functions for water are linear and parallel, with the position of the demand schedule for water shifting to the right as the fixed level of nitrogen fertilizer increases.

The slope of the demand curves indicates the intensity of diminishing returns. If the marginal productivity of water drops

Table 9. Equations of the short-run static demand functions for W when N is fixed at various levels. Colby, Kansas, 1971

Situation number	Level of fixed factor, N (pounds per acre)	
1	50	$W = 20.5754 - 1.895P_W$
2	100	$W = 21.4566 - 1.895P_W$
3	150	$W = 22.3378 - 1.895P_W$
4	200	$W = 23.2189 - 1.895P_W$
5	250	$W = 24.1001 - 1.895P_W$
6	300	$W = 24.9813 - 1.895P_W$

rapidly with greater quantities of water applied, the slope of the demand curve for water will be larger.

The price elasticities of short-run demand functions for water are quite low. If the price of water increases from \$0.60 per acre inch to \$0.80 per acre inch and fertilizer is held constant

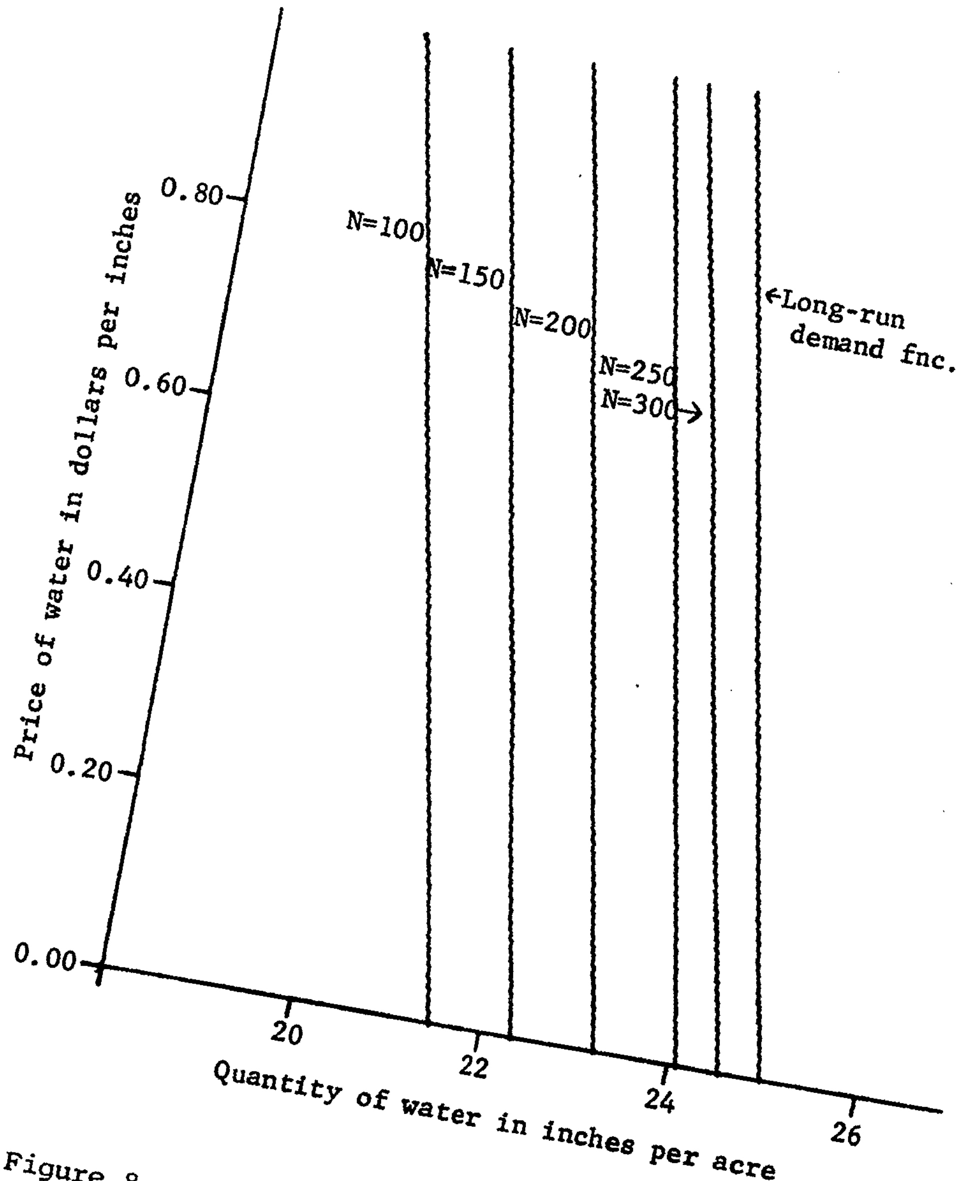

Figure 8. Short-run and long-run demand curves, as estimated from equation (12)

at 200 pounds per acre while the price of corn grain is \$0.024 per pound, the simple arc price elasticity of demand is:

$$E_d = \frac{\Delta W/W}{\Delta P_W/P_W} = \frac{\frac{22.0819 - 21.7029}{22.0819}}{\frac{0.60 - 0.80}{0.80}} = -.051$$

Thus in this price range, if the price of water increases by one percent, the demand for water will decrease by about 0.051 percent.

The static demand function for water may also be computed in a long-run context. The term long run means that the levels of water and nitrogen fertilizer are variable and substitution of one factor for another is possible. This substitution depends on the change in price and the nature of the interaction between water and nitrogen fertilizer.

Equation (11) represents the long-run static demand function for water, derived from the production function (2):

$$W = 24.5375 = 1.9764P_W \tag{11}$$

where nitrogen is not fixed but varies to give the least cost combination of water-nitrogen as the price of water changes. To derive the equation (11) the price of nitrogen and corn are fixed at \$0.8 and \$0.024 per pound respectively.

The slope of the long-run demand for water is slightly less than the slope of any short run demand function. The price elasticity of long-run static demand for water is lower than corresponding short-run elasticity when N is fixed at 250 pounds per acre: namely .0491 and 0.461 for short-run and long-run respectively.

<u>Supply of corn</u>

Static supply functions for a product may also be computed in either a short-run or in a long-run context. Given the underlying technological conditions, whether response in production of corn and use of water might be large or small in relation to price changes can be determined by estimating the supply function. The short-run static supply functions for corn when W is variable, N is fixed at different levels are derived from the production function (11) and presented in Table 10 and shown graphically in Figure 9. The supply curve shifts to the right as the level of fixed factor, N, increases from 50 to 300 pounds per acre. All curves are nearly vertical, indicating that an increase in price of corn would result in negligible changes in supply quantity, i.e., elasticities of supply with respect to corn prices are low.

The estimated supply elasticity of the short-run static supply is about 0.0126, given the price of water $0.60 per acre inch and price of corn is $0.016 per pound. At the price of .024 cents per pound of corn, the elasticity is $0.0056.

The foregoing analysis deals with the short-run static supply curves when water is variable and nitrogen is fixed at specified levels. However, it is quite unlikely that either W or N would

Table 10. Equations of the short-run static supply function for corn when W is variable, N is fixed at different levels. Colby, Kansas, 1971

Situation number	Level of fixed factor, N (pounds per acre)	
1	50	$Y = 5094.4452 - 0.008186P_c^{-2}$
2	100	$Y = 6869.3052 - 0.008186P_c^{-2}$
3	150	$Y = 8241.7374 - 0.008186P_c^{-2}$
4	200	$Y = 9211.7431 - 0.008186P_c^{-2}$
5	250	$Y = 9779.3214 - 0.008186P_c^{-2}$
6	300	$Y = 9944.4724 - 0.008186P_c^{-2}$

be valued along, as combinations are sought to maximize profit. Hence, long-run static supply function, with both W and N variable, is estimated from the production function (11) with the prices of W and N at \$0.60 and \$0.08, respectively.

$$Y = 9946.0087 - 0.0336\ P_C^{-2} \tag{28}$$

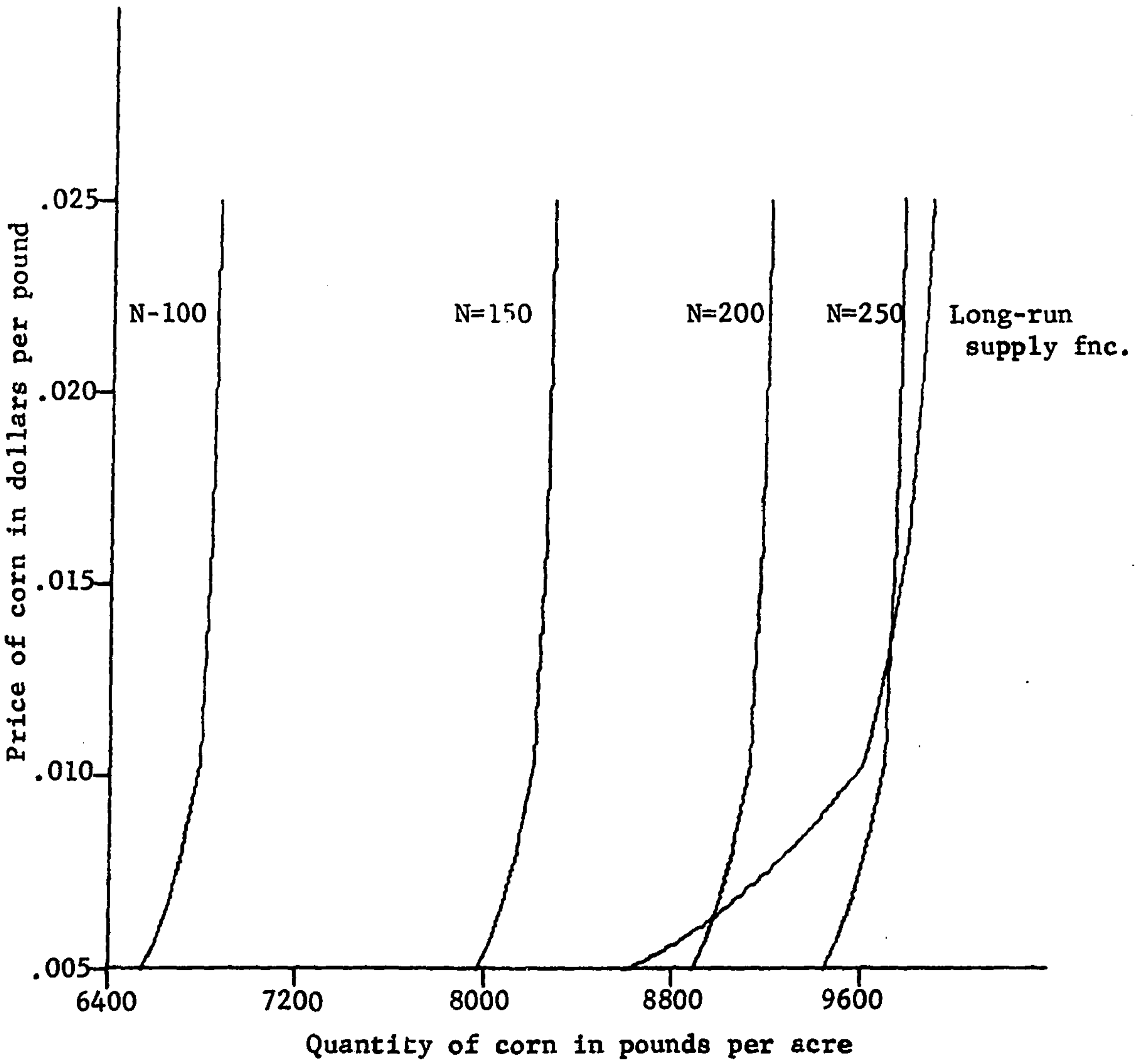

Figure 9. Short-run and long-run static supply curves as estimated from equation (11)

The long-run static supply function has less slope than the derived short-run supply curve for W as variable factor, i.e., the elasticity of the long-run supply curve is greater than for the short-run curve. At the price of $0.024 per pound of corn, the price elasticity is .0118 which is higher than short-run elasticity.

Production Function Analysis and Linear Programming

While the relationship between input and product represented by production function is a physical phenomena, economic principles are involved when any decision is taken in determining or specifying the use of resources. Given a production response function for water, maximization of physical production can be attained when marginal productivity of water is zero. Evaluation of the lowest justifiable water level per acre and marginal productivity of water for various levels of water can be derived from the production function.

If the objective is to maximize profits, the necessary condition is to equate the marginal value product of water to the price of water. The marginal value product of water shows the change in total value of output resulting from a unit change in the quantity of water used per unit of time while the inputs of other resources are held constant. The marginal value product of water varies with different crop enterprises and the relative size of each enterprise.

That is, the marginal value product of some quantity of water used in wheat production will be different from the value of an equal quantity of water used in corn production, and the marginal value product of water used on the same crop may vary with the size of acreage because of its combination with other resources. Furthermore, farm size, the managerial ability of the operator, and the state of the technology employed also influence the marginal value of water.

Given the continuous production function for each crop, if the quantity of water is limited and the objective is to maximize profit, optimum allocation of water can be determined by equating the marginal value product of water for each crop. Assume there are 3 crops and each has a production function of the form $Y = a + bW + bW^2$, and available water is fixed at $\bar{W}$ level. Deriving the marginal value of water for each crop and equating them to the same constant m, the following equation system is obtained.

$$
\begin{aligned}
MVP_1 &= P_1 \frac{dY_1}{dW_1} = m \\
MVP_2 &= P_2 \frac{dY_2}{dW_2} = m \qquad (29) \\
MVP_3 &= P_3 \frac{dY_3}{dW_3} = m \\
\bar{W} &= \Sigma W_i \qquad (i = 1, 2, 3)
\end{aligned}
$$

Solving the system for unknown variables W_1, W_2, W_3 and m, the quantity of available water W_t is allocated such that marginal value productivity of water is equal for each crop and profit from use of the limited water is at a maximum.

According to Heady (1971), given the various crop response function to irrigation water within a farm, equating marginal value productivities with constant prices is largely a physical process. The price of input and output can be considered as weights to allow a common denominator for the various output. In general, a production function does not represent an economic optimization. Instead, it is a partial or suboptimization procedure.

Production in a farm is limited by the resources available, i.e., land, water, fertilizer, man-hour, credit potential, etc. Given the resource constraints, assume that the relevant economic objective is to select from among many alternative production techniques and irrigation levels those that maximize the net return to specified farm resources. Such a problem can be solved by using linear programming which is a very versatile tool that has been used for many years in research in agricultural economics.

Details of linear programming analysis will not be given here. A good description of the theory and application of linear programming is found in Heady and Candler (1958).

In general terms, linear programming is comprised of an objective function of several variables which is maximized (or minimized) subject to a set of constraints on the variables. In order to have an absolute maximum, convexity is assumed for the objective function and constraint set. Basically, four major categories of data are needed to form a linear programming model. The first is there must be possible alternative activities. The second component of linear programming is the knowledge of prices or cost associated with each of the activities. Third, the amount of each resource required by each activity must be known. The fourth and last component is the level of each resource available. Mathematically, linear programming maximizes or minimizes an objective function,

$$Z = \sum_{j=1}^{n} c_j x_j \qquad (j = 1, 2, \ldots n) \tag{30}$$

subject to restraints of the form

$$\sum_{j=1}^{n} a_{ij} X_j \lessgtr b_i \qquad (i = 1, 2, \ldots m) \tag{31}$$

and

$$X_j \geq 0 \tag{32}$$

where

X is net revenue (or cost).

x_j's are the alternative activities.

c_j's are the per-unit prices, net incomes or cost (as the case may be) of the associated activities.

a_{ij}'s are the input-output relationships between the ith resources and the jth activities.

b_i's are the given resource levels or activity restrictions.

The following simplified, hypothetical farm situation is postulated in order to illustrate an application of linear programming and to compare it with conventional production theory.

The estimated technical coefficients, crop prices and available resource levels are given in Table 11. The yield-water relationships in activities P01 and P02 are based on the quadratic function (11) fitted to data for the Colby, Kansas, 1971 experiment. Wheat and sugarbeet information is obtained from Grimes *et al.* (1962) and Hanson (1953), respectively.

The optimal solution for the simplified situation depicted in Table 11, i.e., the combination of inputs and outputs maximizing profits, indicates that 72.2 acres of P01 corn and 210 acres of P03 wheat should be produced. The resources which were limiting were water and capital. If water and capital are increased, the optimal plan would change. P02 was not included in the optimal solution, even though it represents a profit-maximizing activity when considered individually and when resources are not limitational. See paragraph titled "Economic optima". In contrast to production function analysis result, corn should be produced with 18 inches of water instead of 23.4 inches of water.

Table 11. Tableau for simplified linear programming analysis of optimum use-levels for water corn at Colby, Kansas

			Corn grain		Wheat		Sugarbeet	
			P01	P02	P03	P04	P05	P06
Resource availability	B	C	79.3	88.2	44.3	61.7	112.1	146.4
Labor (hours)	2400	R01	10.8	11.6	4.0	4.2	24.0	26.6
Land (acres)	290	R02	1.0	1.0	1.0	1.0	1.0	1.0
Water (inches)	3400	R03	18.0	23.4	10.0	14.3	36.0	48.0
Capital (dollars)	7800	R04	32.4	36.8	26.0	29.2	52.1	64.2

Other applications of linear programming

The present absence of water markets makes it difficult to determine the price of water and the optimum allocation of water among competing users. For most places, allocation of water is in the hands of public agencies. These public agencies must have some criteria for pricing water. The price of water can be estimated by the value of the increase in output resulting from the final unit of water used in production which can be defined as the marginal value of water. Solution to a linear programming problem includes derivation of the "shadow prices" for the resources which limit the solution. If the shadow price or marginal value of resource is positive, it indicates that how much one more unit of a limiting resource would add to total profit.

To estimate the water prices and derive the static-normative demand curve for water, parametric price programming or parametric resource programming can be used. Basically, parametric programming method is a modification of standard linear programming analysis and is discussed in detail in Heady and Candler (1958).

The effect of changing water from zero supply to an amount sufficient to satisfy the requirements for the crop enterprises in the optimal solution can be determined by using the parametric resource programming. Continuous solution of model for increasing water supply level reveals the opportunity cost of water in alternative uses among the various crop enterprises. The estimated marginal value products would indicate the price farmers could afford to pay for a unit of water. Therefore, a demand function for water can be derived showing the quantity of water would be demanded at different water prices.

An estimated static-normative demand function for a farm in Idaho is shown in Figure 10 (Lindeborg, 1970). As price of water (marginal value produce) drops from \$41.34 an acre foot to \$17.70 acre foot, the quantity of water purchased increased from 0 to 625 acre-feet. The profitability of water determines which crops enter the solution. In this example, as price of water drops to \$17.70 acre foot, potato production will be added to sugar beet production. As the price of water drops to \$10.62 an acre foot, hay and grain will be included in the optimum production plan.

The primary objective of this chapter was to estimate the micro-technical relationships among corn grain, water and nitrogen fertilizer. Some of the limitations of this static approach are that estimated production function does not consider the optimal timing of irrigation and fails to include the stochastic nature of precipitation and other pertinent random weather inputs. Stochastic characteristics of water response will be investigated in the following chapters.

In spite of the definite limitations, production function analysis has yielded useful information not otherwise obtainable. Primarily, estimates of production functions are necessary to enumerate the input-output coefficients to be incorporated in farm programming, and best available information for the optimal planning of irrigation projects.

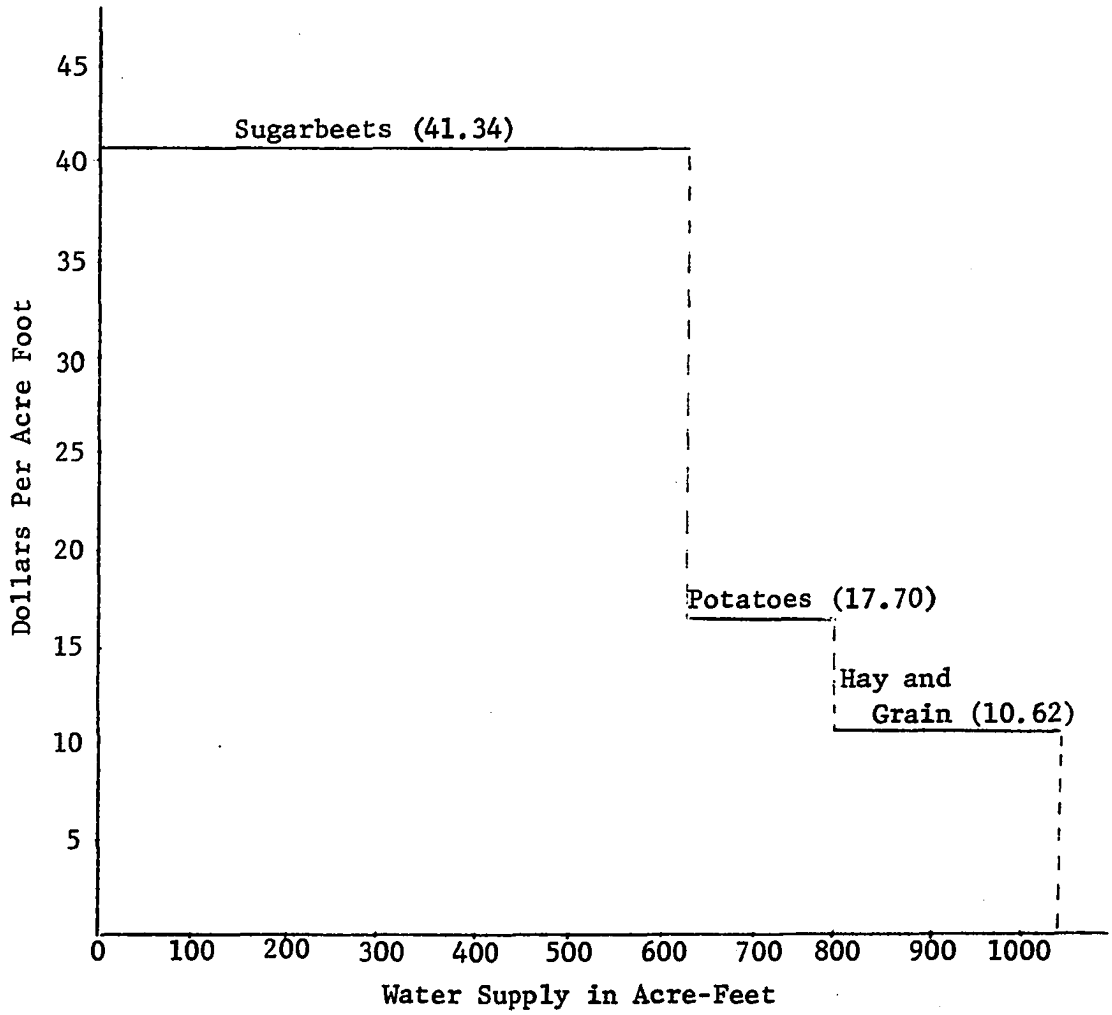

Figure 10. Normative demand curve for water for representative farm in Dry Lake area, Idaho (Lindeborg, 1970)

CHAPTER IV. PLANT-WATER SIMULATION MODEL

In this chapter, a crop-water simulation model of crop response to irrigation will be designated. The model also incorporates the important effects of agronomic, soil and meterological variables. As summarized in Chapter II, the relationship between a plant and the variables affecting its growth is extremely complex. This complex system can, however, be conceptually reduced to a small number of component parts, with each component being related to a group of biological and physical parameters. Given these parameters, development of an operationally-useful simulation model will allow better prediction of crop yield in relation to various inputs.

Review of Literature

Simulation, as a technique of operations research, is now used for a number of problems in agricultural economics as well as other applied fields. Simulation consists of building a model approximating reality which can then be used to investigate the consequences of alternative decisions under varying conditions.

In recent years, several models simulating crop growth have been developed. Soil moisture is generally a primary variable in these models. Most crop-water simulation models are based on the work of Shaw and his associates at Iowa State University (Denmead and Shaw, 1960; Shaw, 1963; Corsi and Shaw, 1971; Shaw and Felch, 1972). No attempt will be made to review these studies. Shaw's simulation model is based on the yield response of corn to varying soil moisture conditions. Shaw demonstrated that his model can be used to satisfactorily explain the reduction in potential yield of corn grain under varying degrees of soil moisture stress at different stages of the growth cycle.

Flinn (1968) was one of the first economists who attempted to estimate a water response function through simulation. According to Flinn, his model provides a rational accounting method for estimating daily soil levels and relating the daily soil moisture level to evaporative parameters in order to obtain an index of plant growth. His simulation results are consistent with actual studies and enable him to estimate the optimal irrigation regime for a given crop under various weather and soil conditions.

Dudley (1970) used a soil moisture-plant growth simulation model similar to that employed by Flinn (1968) to estimate the values of different stages of crop growth in response to specific irrigation strategies. Dudley's main purpose was to generate data for a stochastic, two state variable dynamic programming model to be used for determining optimal intraseasonal allocation of irrigation water.

Anderson and Maass (1971) built a similation model to examine the effects of varying water supply restrictions, water delivery rules, and crop patterns on crop production and farm income in an irrigated area. Use of the simulation model is illustrated by applying it to a problem involving six farms irrigating six different crops, and receiving different proportions of a total available water supply during 14 intervals in an irrigation season. Results show the impact on irrigated agriculture of variation in water supplies over the irrigation season and the importance of the operating procedure used to allocate water. Furthermore, results show that a relatively small change in water supply will have quite different results on individual farms and for the whole irrigated area, depending upon when and where water shortages occur.

Crop-Growth Simulation Model

The soil moisture-plant growth simulation model is developed to estimate the timing and the amount of irrigation water needed to provide adequate soil moisture for optimum yield and to estimate the yield reductions resulting from alternative soil moisture stress conditions. The three main steps in quantifying the simulation model are to: (1) estimate daily values for the factors determining the level of atmospheric demand for moisture by the plant, i.e., actual and potential evapotranspiration; (2) estimate the daily supply of moisture to the crop and its distribution within the root; and (3) estimate the interaction between the demand for and supply of water on economic yield.

Estimating actual and potential evapotranspiration

Both actual and potential evapotranspiration are estimated within the simulation model by using the following relationships:

$$E_t = f \cdot E_o \tag{33}$$

$$E_a = P \cdot E_t \tag{34}$$

Where

E_t = potential evapotranspiration

f = crop factor

E_o = daily free water evaporation (pan evaporation)

E_a = actual evapotranspiration

P = soil moisture factor.

Definition and discussion of the parameters and variables in equations (33) and (34) are given below.

<u>Potential evapotranspiration</u> (E_t) Potential evapotranspiration has been defined as the sum of water vapor evaporation from the soil-air interface when the soil moisture level is at field capacity conditions and from plants which completely cover the ground surface. Among the various prediction methods for estimating potential evapotranspiration are the Blaney-Criddle (1950), Thorntwaite (1948), and Penman (1948) methods. Results of Shaw's (1963) studies indicate that the Penman method is the most accurate of the three when tested under Iowa conditions. Penman's method is essentially equation (33). That is, daily potential evapotranspiration is estimated as the daily free water evaporation multiplied by a crop factor, f.

Crop factor (f) Research shows a close relationship between the rate of consumptive use by a crop (evapotranspiration) and the rate of evaporation from a free water surface (pan evaporation). The crop factor is defined as the ratio of potential evaporation to the actual evaporation from a free water surface. Shaw (1963) reports that during the early stage of corn growth, at a time plants are small and provide little ground cover, the measured consumptive use is considerably lower than the average free water evaporation, i.e., f is relatively small for these stages. As the rate of corn growth increases, so does the value of f. After silking, however, the ratio starts decreasing. The relationship between the ratio of evapotranspiration to pan evaporation and the plant growth cycle is shown in Figure 11. The relationships in Figure 11 are based on Shaw's studies of corn in Iowa.

Free water evaporation (E_o) Equation (33) indicates that estimation of potential evapotranspiration also requires estimation of free water evaporation. Penman (1948) showed that E_o depends on temperature, radiation, wind and humidity. Shaw (1963) suggests that whenever there is a shortage of data, pan evaporation readings can be used to approximate free water evaporation.

Actual evapotranspiration (E_a) The supply of water to a crop depends not only on available soil moisture but also on the depth and density of the plant's root zone. Where available soil moisture in the root zone drops to a point where the plant can no longer extract sufficient moisture to meet its transpiration needs,

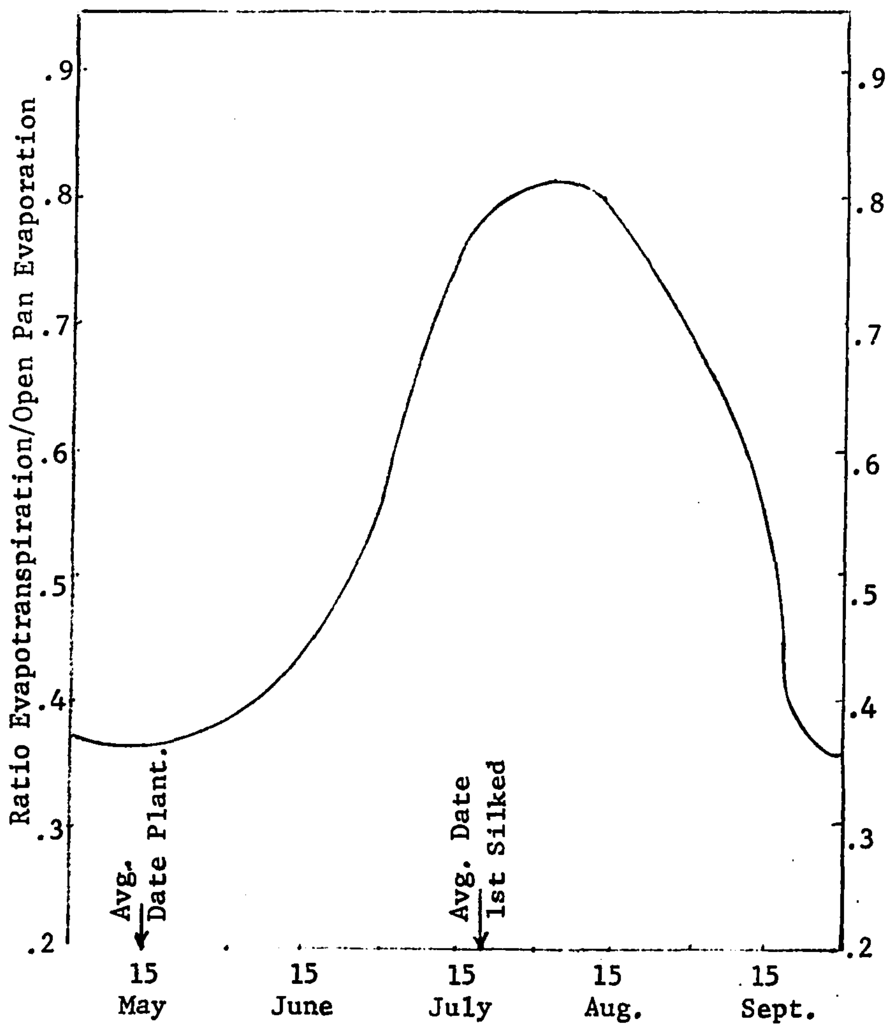

Figure 11. Ratio of evapotranspiration of corn to open-pan evaporation throughout the growing season (Shaw, 1963)

actual evapotranspiration falls below potential evapotranspiration. For each soil-plant complex, there is a maximum rate at which the plant can extract water from the soil. This rate is called the maximum intake rate (Em). The level of Em depends on the prevailing soil moisture condition while E_t depends on climatic conditions.

Actual evapotranspiration equals the potential rate when the maximum intake rate, Em, exceeds the potential evapotranspiration rate, E_t. If, however, the maximum intake rate is less than the potential evapotranspiration rate, actual evapotranspiration will be equal to the maximum intake rate. The relationships among E_a, E_t and Em are summarized in equations (35) and (36). Both equations (35) and (36) can be utilized to predict actual evapotranspiration.

$$E_a = E_t \quad \text{if} \quad E_t < Em \tag{35}$$

and

$$E_a = Em \quad \text{if} \quad E_t > Em \tag{36}$$

Soil moisture factor (P) Fleming (1964) has demonstrated that the relationships in equations (35) and (36) can be combined. See equation (37). P is termed the "soil moisture factor" and

$$P = \frac{E_a}{E_t} = f\ (E_t,\ ASM) \tag{37}$$

ASM is the available soil moisture. As long as E_a equals E_t, P equals 1.0. When E_a is less than E_t, i.e., $E_a = E_m$, P is less than 1.0. Empirical values of P can be estimated for different soil moisture and atmospheric demand levels. Figure 12a demonstrates the relative transpiration rate (i.e., the ratio of actual to potential evapotranspiration) of corn as a function of the prevailing soil moisture level and daily potential transpiration while based on studies of Denmead and Shaw (1962).

<u>Estimating the supply of soil moisture to a crop</u>

The soil moisture content in the root zone at the end of any day, SM_t, is estimated in equation (38) where:

$$SM_t = SM_{t-1} + R_t + I_t + AW_t - E_{at} - DR_t \tag{38}$$

subject to $PWP \leq SM \leq FC$

SM_t = available soil moisture level on day t

SM_{t-1} = available soil moisture level at the end of day t-1

R_t = effective rainfall on day t

I_t = irrigation water applied on day t

AW_t = available soil water in the root zone on day t

E_{at} = actual evapotranspiration on day t

DR_t = deep percolation and runoff on day t

PWP = permanent wilting point

FC = field capacity.

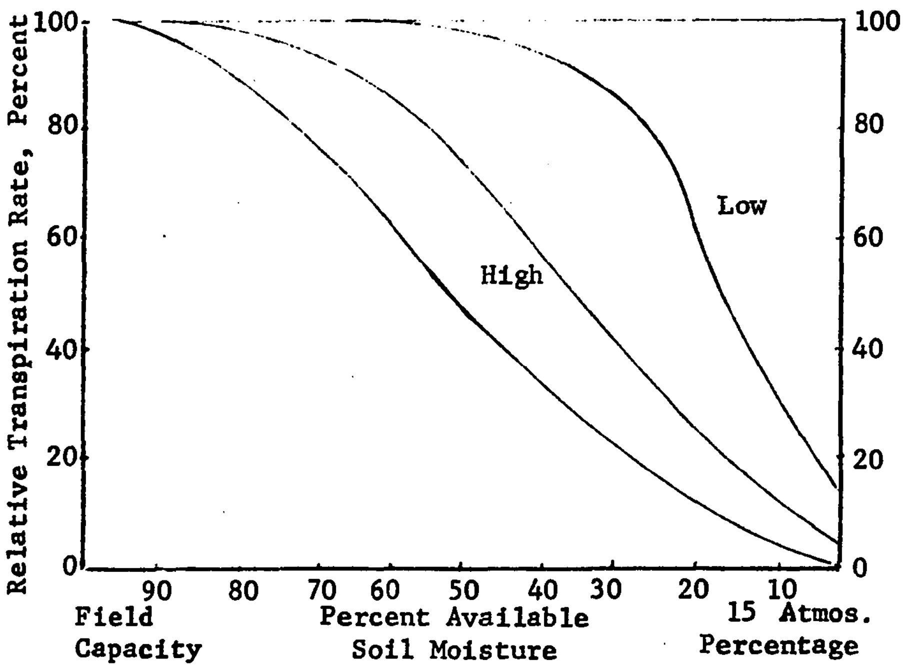

Figure 12a. Relative transpiration rates for different atmospheric demand intensities and soil water content (Denmead and Shaw, 1962)

This constraint implies that available soil moisture will be allowed to deplete to the permanent wilting point and will not exceed the field capacity of soil.

Estimating crop growth

Many biologists have shown that moisture shortage, i.e., stress, at different stages of crop development may have differential effects on harvested yield. Further, stress in one particular stage will likely affect subsequent growth and yield. Since there is little empirical evidence to quantify these interrelationships, the effects of stress on harvested yield in each stage of crop growth are generally assumed to be independent. This is, of course, a simplifying assumption.

Net growth on any day is related to the occurrence of moisture stress on that day. As noted earlier, moisture stress occurs whenever evapotranspiration reduces available soil moisture to that level which is insufficient to allow the plant to grow at its normal rate. In other words, crop growth ceases where actual evapotranspiration falls below potential evapotranspiration. In Dudley's simulation model (1969), an important assumption was that plants grow at their potential rate on any day when daily $E_a = E_t$ and $P = 1.0$. No growth occurs on days when $E_a < E_t$ and $P < 1.0$. The implication of this assumption is that the effect of moisture stress at different stages of crop development would be the same on harvested yield.

In this study, a crop growth stage weighting modification is incorporated into the simulation model. Morris (1972) estimated some weights to evaluate the effect of stress on potential yield for different stages. Weights for each day were obtained from the relationship (100 - WT/100) where 100 represents the relative yield of unstressed plants and WT is the relative yield for plants stressed during the appropriate periods. The weights are presented in Figure 12b, but they were entered in the model in tabular form for each of the 135 days of the growing period.

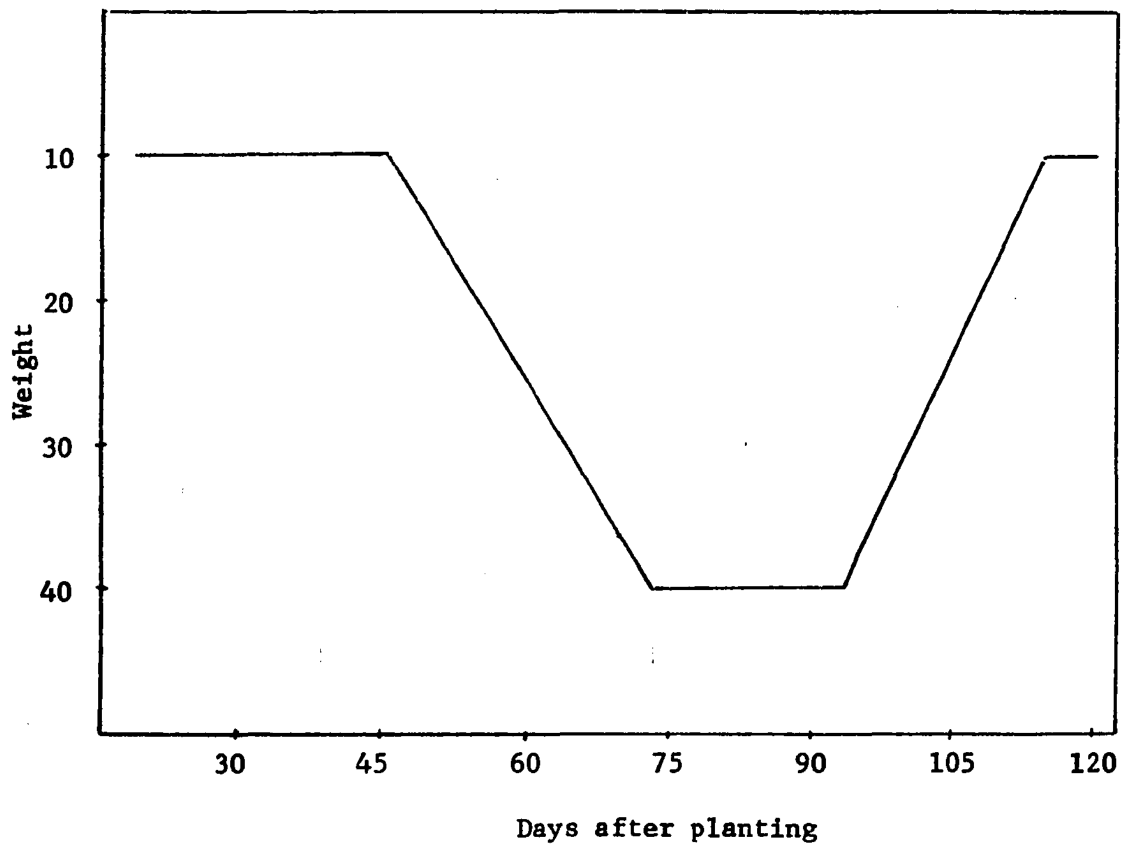

Figure 12b. Estimated weights for crop stress during the growing periods (Morris, 1972)

Basic Data

Some of the variables required for the simulation model such as water holding capacity and infiltration rates of soils are easily measured in the field. Climatological and pan evaporation data are often readily available from weather stations. In those cases where specific values of parameters such as f and P are not available, it is necessary to derive estimates of these by analogy with other locations and known biological relationships.

In this study, the various parameters and assumptions necessary for quantifying the simulation model are discussed and derived in relation to conditions existent at Colby, Kansas.

Free water evaporation (E_o)

E_o was estimated by using class A pan evaporation data from the Colby Experiment Station for 1971. According to Shaw (1963), evaporation pans have a different type of surface than does a crop cover. Consequently, the pan data should be adjusted for surface temperature. Due to the limited data available for this study, pan evaporation data are assumed to approximate free water evaporation.

Crop coefficient (f)

As stated above, the crop coefficient relates potential evapotranspiration to free water evaporation, and its value depends on the stage of crop growth, plant height and genotype. Since empirical observations of f for the Colby area do not exist, a modification of the f values used by Shaw (1963) was considered necessary. When relatively higher temperatures and dry weather occur, it is assumed that the ratio of potential evapotranspiration to pan evaporation, i.e., f, will be higher. The f values for various stages of corn growth at Colby, Kansas, as presented in Table 15, column 3.

Soil factor (P)

Since estimates of P are not available for the Colby area, values for other locations have been taken and modified as a proxy for conditions in Western Kansas. Denmead and Shaw (1962) measured P as a function of the prevailing soil moisture level and daily potential evapotranspiration. As shown in Figure 12 above, under conditions of high atmospheric demand, transpiration from a plant will decrease at a relatively high moisture content because the plant cannot supply water fast enough to meet the high demand. At low atmospheric demand, no reduction in water loss will occur until relatively low soil-moisture content is reached. With these relationships in mind and taking into account the relatively high atmospheric demand at the Colby site, the modified values of P are presented in Table 12.

Rainfall

Several assumptions have been made concerning rainfall. First, because light showers are largely intercepted by crop foliage or only wet a thin surface layer of the soil, rainfall less than .10

Table 12. P values for 2 range of soil moisture and atmospheric condition[a]

Available soil moisture %	Level of daily potential transpiration Low (<.30")	High (>.30")
100-90	1.00	1.00
89-80	1.00	1.00
79-70	.97	.89
69-60	.94	.77
59-50	.84	.63
49-40	.74	.48
39-30	.56	.34
29-20	.40	.22
19-0	.24	.16

[a]Source: Modified from Denmead and Shaw (1962).

inches is ignored. Secondly, the rainfall which actually enters the soil is assumed to be 90 percent of recorded rainfall until the soil moisture reaches field capacity. Thirdly, when rainfall is insufficient to raise the upper 6 to 8 inches of soil to field capacity, this moisture will be held near the surface of soil and will be used at a rate of potential evapotranspiration. According to Dudley (1970), this is a realistic assumption because the upper 6 to 8 inches of soil constitutes the zone of maximum root concentration. Further, evaporation will remove moisture from the top soil at the potential evapotranspiration rate at least until the soil moisture content of the upper layer equals that prevailing throughout the remainder of root zone. Fourthly, rainfall in excess of field capacity will be lost through runoff or deep percolation.

Evapotranspiration zone

The evapotranspiration zone is defined as the depth of soil from which soil moisture is being extracted by the crop roots. Generally, the volume of water available to the crop on any day depends on the water holding capacity of soil and the depth of the evapotranspiration zone.

In this study, it is assumed that the soil is homogeneous and has a constant water holding capacity of about 2.43 inches per foor throughout the root zone. Actual and assumed water holding capacities of the Keith Silt Loam soil at the Colby site are presented in Table 13.

With respect to the root depth, Shaw's (1963) assumption is used. Shaw assumed that the root zone will be 6 inches for the first 22 days after planting. Thereafter a linear expansion of the crop root zone of 6 inches per week occurs until a maximum depth of 5 feet is reached nine weeks after planting. That is, corn reaches its maximum rooting depth of 5 feet in eighty-five

Table 13. Actual and assumed water holding capacities of Keith Silt loan soil, Colby, Kansas

Depth (feet)	Actual water holding capacity (inches)	Assumed water holding capacity (inches)
1	2.36	2.43
2	2.69	2.43
3	2.44	2.43
4	2.38	2.43
5	2.28	2.43

days after planting. In terms of the water holding capacity (WHC) of the evapotranspiration zone, the WHC of top 6 inches of soil is 1.215 inches during the first 22 days after planting. During the next 63 days, the WHC of the root zone will increase at a constant rate of .173571 inches per day until it reaches the field capacity of 12.15 inches for the 5 foot root zone.

Actual and potential yield

To evaluate the contribution of plant growth on each day, Flinn and Musgrave (1967) plotted a potential and actual growth curves of an annual crop under investigation. Potential growth can be defined as the growth that can be obtained when the soil moisture is adequate throughout the growing season. Whenever soil moisture stress occurs, potential yield is reduced, as indicated by curves 2 and 3, in Figure 13. The duration and severity of the stress will have different effects on final yield. Time of the occurrence of stress is also important during the growth cycle. If stress occurs in earlier stages of growth, as suggested by Curve 2, the yield reduction will be relatively smaller than stress occurring during the rapid growth period, as indicated by Curve 3.

As an approximation, potential yield in each stage of the growing season is weighted by assigning the proportion of potential growth in each stage as it relates to final potential yield.

In this study, Hanway's (1963) accumulated growth curve is used to estimate the daily growth. The growth season is divided into 10 stages. The percentage contribution of each stage to the potential yield is listed in Table 14.

Given the daily yield, a monetary value of this yield will be estimated by using the following procedure. Assume, for example, the price of corn is \$0.024 per pound and the potential yield is 9,200 pounds. Given a 2% contribution to final yield by the 14 days of plant growth in Stage 2 and assuming no stress days occur, the monetary value of yield for that 14 day period would be:

$$(9200)\ (.024)\ (2\ \text{percent}) = \$4.416$$

and the value of a day's growth would be:

$$\$.416 \div 14 = \$0.2944$$

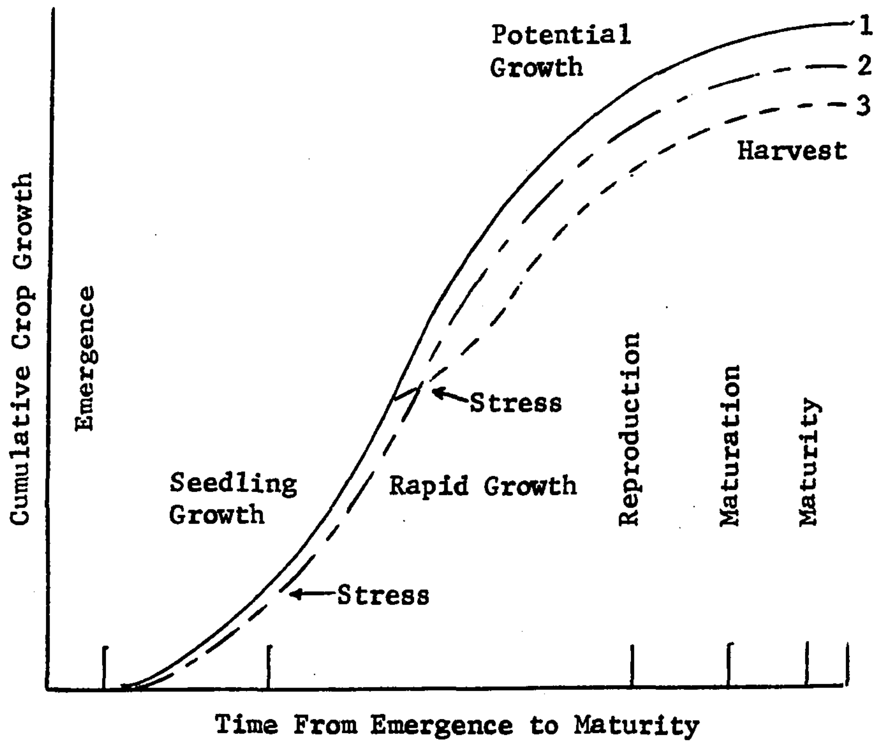

Figure 13. Theoretical potential and actual crop curves of an annual crop (Flinn and Musgrave, 1967)

Table 14. Contribution of each growth stage to potential corn yield

Growth stages	Days after planting	Percentage of contribution of each stage to potential yield
1	23	0
2	37	2
3	51	8
4	65	18
5	73	15
6	87	16
7	99	16
8	111	18
9	123	6
10	135	1

Irrigation Decision Rule

The first basic assumption in the simulation model with respect to irrigation water application is that the quantity of water applied per irrigation is always that needed to return soil moisture in the whole root zone to the field capacity level. Irrigation timing decisions consist of daily decisions to irrigate or postpone depending on the state of many influencing variables.

The criterion generally used is to apply irrigation water whenever the existing available soil moisture at the beginning of day t plus the expected net effect of evapotranspiration and rainfall on day t is depleted to a prescribed minimum level. Given this criterion, the decision maker can predict the rainfall and evapotranspiration at the start of a day and determine how much irrigation water should be applied. In the following computerized simulation model, irrigation water is applied whenever the available soil moisture in the root zone is below 70 percent of field capacity, taking into account the predicted rainfall and evapotranspiration which would occur on individual days.

The Computerized Simulation Model

The computer program is written by Dudley (1970). Some modifications had to be done to transform the program from Fortran II-D computer language to Fortran WATIV to run it on an IBM 360/65 digital computer. The program is presented in Appendix A.

Table 15 presents a sample of the computer output generated by the corn growth simulation model for Colby, Kansas, 1971. The irrigation decision rule was to irrigate whenever soil moisture levels fall to 70 percent of field capacity during the 90-day irrigation season (June 20-September 19).

A short explanation of the sample of output is presented below. Column 1 identifies successive days of the plant growth season. The irrigation season is a subset of the growth season. The

Table 15. Computer output of corn-growth simulation for Colby, Kansas, 1971

Day no. (1)	Pan evap. (in.) (2)	F (3)	E_t (4)	P (5)	E_a (6)	Rain (in.) (7)	Rain bal. (in.) (8)	Soil water (%) (9)	Irgn. (in.) (10)	Evtn. zone (in.) (11)	Grs. rev. ($) (12)
1	0.06	0.09	0.005	1.00	0.005	0.00	0.000	100.000	0.000	1.220	0.00
2	0.26	0.09	0.023	1.00	0.023	0.00	0.000	98.082	0.000	1.220	0.00
3	0.30	0.09	0.027	1.00	0.027	0.00	0.000	95.869	0.000	1.220	0.00
4	0.15	0.09	0.013	1.00	0.013	0.22	0.000	100.000	0.000	1.220	0.00
5	0.02	0.09	0.002	1.00	0.002	0.49	0.000	100.000	0.000	1.220	0.00
6	0.17	0.09	0.015	1.00	0.015	0.00	0.000	100.000	0.000	1.220	0.00
7	0.26	0.09	0.023	1.00	0.023	0.00	0.000	98.082	0.000	1.220	0.00
8	0.30	0.09	0.027	1.00	0.027	0.00	0.000	95.869	0.000	1.220	0.00
9	0.43	0.10	0.043	1.00	0.043	0.00	0.000	92.344	0.000	1.220	0.00
10	0.46	0.11	0.051	1.00	0.051	0.21	0.000	100.000	0.000	1.220	0.00
11	0.34	0.12	0.041	1.00	0.041	0.00	0.000	100.000	0.000	1.220	0.00
12	0.73	0.12	0.088	1.00	0.088	0.90	0.000	100.000	0.000	1.220	0.00
13	0.06	0.13	0.008	1.00	0.008	0.32	0.000	100.000	0.000	1.220	0.00
14	0.20	0.14	0.028	1.00	0.028	0.00	0.000	100.000	0.000	1.220	0.00
15	0.29	0.14	0.041	1.00	0.041	0.00	0.000	96.672	0.000	1.220	0.00
16	0.38	0.14	0.053	1.00	0.053	0.00	0.000	92.311	0.000	1.220	0.29
17	0.30	0.15	0.045	1.00	0.045	0.00	0.000	88.623	0.000	1.220	0.29
18	0.34	0.15	0.051	1.00	0.051	0.00	0.000	84.442	0.000	1.220	0.29
19	0.29	0.16	0.046	1.00	0.046	0.00	0.000	80.639	0.000	1.220	0.29
20	0.36	0.16	0.058	0.00	0.058	0.00	0.000	75.918	0.000	1.220	0.29
21	0.32	0.16	0.051	0.89	0.046	0.00	0.000	72.183	0.000	1.220	0.26
22	0.19	0.17	0.032	0.97	0.031	0.00	0.000	69.614	0.000	1.220	0.26
23	0.11	0.18	0.020	0.97	0.019	0.00	0.000	72.019	0.000	1.393	0.26
24	0.01	0.19	0.002	1.00	0.002	0.53	0.000	100.000	0.000	1.567	0.29
25	0.38	0.20	0.076	1.00	0.076	0.00	0.000	100.000	0.000	1.740	0.29
26	0.34	0.22	0.075	1.00	0.075	0.14	0.000	100.000	0.000	1.914	0.29
27	0.10	0.24	0.024	1.00	0.024	0.00	0.000	100.000	0.000	2.037	0.29

28	0.30	0.27	0.081	1.00	0.081	0.00	0.000	96.417	0.000	2.261	0.29
29	0.22	0.30	0.066	1.00	0.066	0.12	0.000	98.891	0.000	2.434	0.29
30	0.40	0.31	0.124	1.00	0.124	0.00	0.000	94.210	0.000	2.608	0.29
31	0.39	0.32	0.125	1.00	0.125	0.00	0.000	90.084	0.000	2.781	1.18
32	0.38	0.34	0.129	1.00	0.129	0.00	0.000	86.294	0.000	2.955	1.18
33	0.34	0.37	0.126	1.00	0.126	0.00	0.000	83.033	0.000	3.123	1.18
34	0.27	0.38	0.103	1.00	0.103	0.13	0.027	84.754	0.000	3.302	1.18
35	0.30	0.39	0.117	1.00	0.117	0.15	0.060	86.464	0.000	3.475	1.18
36	0.44	0.40	0.176	1.00	0.176	0.31	0.039	90.780	0.000	3.649	1.18
37	0.40	0.41	0.164	1.00	0.164	0.00	0.000	86.908	0.000	3.822	1.18
38	0.37	0.42	0.155	1.00	0.155	0.00	0.000	83.587	0.000	3.996	1.18
39	0.38	0.43	0.163	1.00	0.163	0.00	0.000	80.351	0.000	4.169	1.18
40	0.25	0.43	0.107	1.00	0.107	0.79	0.205	96.851	0.000	4.343	1.18
41	0.38	0.44	0.167	1.00	0.167	0.00	0.000	93.270	0.000	4.516	1.18
42	0.43	0.44	0.189	1.00	0.189	0.00	0.000	89.485	0.000	4.690	1.18
43	0.59	0.45	0.265	1.00	0.265	0.00	0.000	84.401	0.000	4.863	1.18
44	0.64	0.46	0.294	1.00	0.294	0.00	0.000	79.093	0.000	5.037	1.18
45	0.43	0.47	0.202	0.89	0.180	0.00	0.000	76.337	0.000	5.210	1.06
46	0.45	0.48	0.216	0.89	0.192	0.00	0.000	73.529	0.000	5.384	2.36
47	0.51	0.49	0.250	0.89	0.222	0.00	0.000	70.353	0.000	5.557	2.33
48	0.45	0.50	0.225	0.89	0.200	0.00	0.000	100.000	1.648	5.731	2.31
49	0.56	0.51	0.286	1.00	0.286	0.00	0.000	100.000	0.000	5.904	2.65
50	0.58	0.52	0.302	1.00	0.302	0.00	0.000	95.038	0.000	6.078	2.65
51	0.70	0.53	0.371	1.00	0.371	0.00	0.000	89.241	0.000	6.251	2.65
52	0.34	0.54	0.454	1.00	0.454	0.00	0.000	82.471	0.000	6.425	2.65
53	0.94	0.55	0.517	1.00	0.517	0.00	0.000	75.097	0.000	6.598	2.65
54	0.63	0.56	0.353	0.89	0.314	0.00	0.000	71.098	0.000	6.772	2.15
55	0.58	0.57	0.331	0.89	0.294	0.00	0.000	100.000	1.957	6.945	2.12
56	0.41	0.58	0.238	1.00	0.238	0.67	0.000	100.000	0.000	7.119	2.65
57	0.39	0.59	0.230	1.00	0.230	0.00	0.000	100.000	0.000	7.292	2.65
58	0.40	0.60	0.240	1.00	0.240	0.00	0.000	96.785	0.000	7.466	2.65
59	0.46	0.61	0.281	1.00	0.281	0.46	0.000	99.207	0.000	7.639	2.65
60	0.55	0.62	0.341	1.00	0.341	0.51	0.000	100.000	0.000	7.813	2.65
61	0.36	0.64	0.230	1.00	0.230	0.00	0.000	100.000	0.000	7.986	3.01
62	0.45	0.66	0.297	1.00	0.297	0.00	0.000	96.360	0.000	8.160	3.01
63	0.53	0.68	0.360	1.00	0.360	0.00	0.000	92.111	0.000	8.333	3.01

Table 15. (continued)

Day no.	Pan evap. (in.)	F	E_t	P	E_a	Rain (in.)	Rain bal. (in.)	Soil water (%)	Irgn. (in.)	Evtn. zone (in.)	Grs. rev. ($)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
64	0.14	0.69	0.097	1.00	0.097	0.40	0.029	95.839	0.000	8.507	3.01
65	0.36	0.70	0.252	1.00	0.252	0.00	0.000	93.019	0.000	8.680	3.01
66	0.44	0.72	0.317	1.00	0.317	0.00	0.000	89.577	0.000	8.854	3.01
67	0.39	0.73	0.285	1.00	0.285	0.00	0.000	86.624	0.000	9.027	3.01
68	0.44	0.75	0.330	1.00	0.330	0.00	0.000	83.289	0.000	9.201	3.01
69	0.45	0.76	0.342	1.00	0.342	0.00	0.000	79.950	0.000	9.374	3.01
70	0.31	0.78	0.242	1.00	0.242	0.00	0.000	77.782	0.000	9.548	3.01
71	0.44	0.80	0.352	0.89	0.313	0.00	0.000	74.956	0.000	9.721	1.81
72	0.47	0.81	0.381	0.89	0.339	0.00	0.000	71.971	0.000	9.895	1.66
73	0.58	0.82	0.476	0.89	0.423	0.00	0.000	100.000	2.773	10.068	1.63
74	0.55	0.84	0.462	1.00	0.462	0.00	0.000	100.000	0.000	10.242	2.72
75	0.32	0.85	0.272	1.00	0.272	0.00	0.000	97.388	0.000	10.415	2.72
76	0.55	0.86	0.473	1.00	0.473	0.00	0.000	92.964	0.000	10.589	2.72
77	0.45	0.87	0.391	1.00	0.391	0.00	0.000	89.440	0.000	10.762	2.72
78	0.47	0.88	0.414	1.00	0.414	0.00	0.000	85.826	0.000	10.936	2.72
79	0.41	0.89	0.365	1.00	0.365	0.00	0.000	82.762	0.000	11.109	2.72
80	0.44	0.90	0.396	1.00	0.396	0.00	0.000	79.518	0.000	11.283	2.72
81	0.50	0.90	0.450	0.89	0.400	0.00	0.000	76.332	0.000	11.456	1.63
82	0.21	0.91	0.191	0.97	0.185	0.00	0.000	75.091	0.000	11.630	1.63
83	0.45	0.92	0.414	0.89	0.368	0.00	0.000	72.336	0.000	11.803	1.63
84	0.40	0.93	0.372	1.00	0.372	0.37	0.000	72.720	0.000	11.977	2.72
85	0.23	0.93	0.214	0.97	0.207	0.00	0.000	71.401	0.000	12.150	1.63
86	0.37	0.93	0.344	0.89	0.306	0.00	0.000	100.000	3.475	12.150	1.63
87	0.39	0.92	0.359	1.00	0.359	0.00	0.000	100.000	0.000	12.150	2.72
88	0.39	0.91	0.355	1.00	0.355	0.00	0.000	97.079	0.000	12.150	2.72
89	0.26	0.90	0.234	1.00	0.234	0.21	0.000	96.882	0.000	12.150	2.72

90	0.41	0.90	0.369	1.00	0.369	0.00	0.000	93.845	0.000	12.150	2.72
91	0.43	0.90	0.387	1.00	0.387	0.00	0.000	90.660	0.000	12.150	2.72
92	0.42	0.90	0.378	1.00	0.378	0.00	0.000	87.548	0.000	12.150	2.72
93	0.30	0.89	0.267	1.00	0.267	0.00	0.000	85.351	0.000	12.150	2.72
94	0.30	0.88	0.264	1.00	0.264	0.00	0.000	83.178	0.000	12.150	2.72
95	0.38	0.87	0.331	1.00	0.331	0.00	0.000	80.457	0.000	12.150	2.72
96	0.35	0.86	0.301	1.00	0.301	0.00	0.000	77.980	0.000	12.150	2.72
97	0.45	0.85	0.382	0.89	0.340	0.00	0.000	75.178	0.000	12.150	1.74
98	0.45	0.84	0.378	0.89	0.336	0.00	0.000	72.409	0.000	12.150	2.15
99	0.59	0.83	0.490	0.89	0.436	0.00	0.000	100.000	3.352	12.150	2.19
100	0.44	0.82	0.361	1.00	0.361	0.00	0.000	100.000	0.000	12.150	3.31
101	0.24	0.82	0.197	1.00	0.197	0.00	0.000	98.380	0.000	12.150	3.31
102	0.26	0.81	0.211	1.00	0.211	0.00	0.000	96.647	0.000	12.150	3.31
103	0.42	0.80	0.336	1.00	0.336	0.00	0.000	93.882	0.000	12.150	3.31
104	0.45	0.79	0.355	1.00	0.355	0.00	0.000	90.956	0.000	12.150	3.31
105	0.48	0.76	0.374	1.00	0.374	0.00	0.000	87.874	0.000	12.150	3.31
106	0.22	0.77	0.169	1.00	0.169	0.00	0.000	86.480	0.000	12.150	3.31
107	0.41	0.76	0.312	1.00	0.312	0.00	0.000	83.916	0.000	12.150	3.31
108	0.45	0.75	0.337	1.00	0.337	0.00	0.000	81.138	0.000	12.150	3.31
109	0.41	0.74	0.303	1.00	0.303	0.13	0.000	79.711	0.000	12.150	3.31
110	0.42	0.73	0.307	0.89	0.273	0.00	0.000	77.465	0.000	12.150	0.85
111	0.45	0.72	0.324	0.89	0.288	0.00	0.000	75.092	0.000	12.150	0.87
112	0.47	0.71	0.334	0.89	0.297	0.00	0.000	72.647	0.000	12.150	0.89
113	0.46	0.70	0.322	0.89	0.287	0.00	0.000	70.289	0.000	12.150	0.91
114	0.52	0.69	0.359	0.89	0.319	0.00	0.000	100.000	3.610	12.150	0.92
115	0.40	0.68	0.272	1.00	0.272	0.00	0.000	100.000	0.000	12.150	1.02
116	0.45	0.67	0.301	1.00	0.301	0.00	0.000	97.519	0.000	12.150	1.02
117	0.53	0.67	0.355	1.00	0.355	0.48	0.000	98.546	0.000	12.150	1.02
118	0.40	0.65	0.260	1.00	0.260	0.00	0.000	96.407	0.000	12.150	1.02
119	0.41	0.64	0.262	1.00	0.262	0.00	0.000	94.247	0.000	12.150	1.02
120	0.54	0.63	0.340	1.00	0.340	0.00	0.000	91.447	0.000	12.150	1.02
121	0.56	0.61	0.342	1.00	0.342	0.00	0.000	83.636	0.000	12.150	1.02
122	0.56	0.60	0.336	1.00	0.336	0.00	0.000	85.870	0.000	12.150	1.02
123	0.42	0.59	0.248	1.00	0.248	0.00	0.000	83.831	0.000	12.150	0.17

Table 15. (continued)

Day no. (1)	Pan evap. (in.) (2)	F (3)	E_t (4)	P (5)	E_a (6)	Rain (in.) (7)	Rain bal. (in.) (8)	Soil water (%) (9)	Irgn. (in.) (10)	Evtn. zone (in.) (11)	Grs. rev. ($) (12)
124	0.49	0.56	0.274	1.00	0.274	0.00	0.000	81.572	0.000	12.150	0.17
125	0.56	0.55	0.308	1.00	0.308	0.00	0.000	79.038	0.000	12.150	0.17
126	0.28	0.53	0.148	0.97	0.144	0.00	0.000	77.853	0.000	12.150	0.15
127	0.40	0.52	0.208	0.89	0.185	0.00	0.000	76.329	0.000	12.150	0.15
128	0.51	0.50	0.255	0.89	0.227	0.00	0.000	74.461	0.000	12.150	0.15
129	0.45	0.48	0.216	0.89	0.192	0.00	0.000	72.879	0.000	12.150	0.15
130	0.56	0.46	0.258	0.89	0.229	0.00	0.000	70.992	0.000	12.150	0.15
131	0.42	0.45	0.189	0.89	0.168	0.00	0.000	100.000	3.525	12.150	0.15
132	0.42	0.44	0.185	1.00	0.185	0.00	0.000	100.000	0.000	12.150	0.17
133	0.28	0.43	0.120	1.00	0.120	0.00	0.000	99.009	0.000	12.150	0.17
134	0.04	0.42	0.017	1.00	0.017	0.16	0.000	100.000	0.000	12.150	0.17
135	0.04	0.41	0.016	1.00	0.016	0.52	0.000	100.000	0.000	12.150	0.17

irrigation treatments are started 45 days after the planting day. This assumption is consistent with the actual experiment conducted at Colby, Kansas. Column 2 records daily class A pan evaporation readings, measured in inches. Column 3 lists the daily values of the crop factor, f, which relates the pan evaporation to potential evaporation as discussed above. That is, Column 3 = Column 4 ÷ 2. Column 4 shows the estimated values of potential evaporation, E_t, measured in inches per day. It is derived from Columns 2 and 3 by applying the relationship $E_t = f.E_o$.

Column 5 estimates the soil factor, P. P is a function of daily free water evaporation shown in Column 2 and the available soil moisture in the root zone listed in Column 9. Values of P for various soil moisture and atmospheric conditions are read in the program.

Column 6 shows the estimates of the actual evapotranspiration, E_a, measured in inches per day. The estimates are derived from Volumns 4 and 5 by using the relationship $E_a = P.E_t$.

Column 7 lists the additions of effective rainfall to soil moisture, measured in inches per day. The effective rainfall is discussed in detail above. Column 8 shows the quantity of available soil moisture held in the upper root zone (6 to 8 inches) after the rainfall. This is called "rain balance". See paragraph titled "Rainfall".

Column 9 estimates the available soil moisture percentage within the root zone. The estimate is a function of rooting depth, daily evapotranspiration, rainfall, and irrigation. When available soil moisture is at 100 percent, soil moisture is at the field capacity level.

Column 10 indicates the timing of irrigations and the quantity of water applied each time when the available soil moisture in Column 9 falls to 70 percent of field capacity. Column 11 shows the water-holding capacity of the root zone after root zone extension is accounted for.

Column 12 estimates the monetary value of daily growth of corn grain. The value of P (Column 5) is used to determine whether the value of corn growth on any day is zero or equal to the relevant value of corn growth which is read in as data.

As shown in Column 10, irrigation treatments take place on days 48, 55, 73, 81, 99, 114 and 131. The irrigations occurred on days when the available soil moisture fell to 70 percent of field capacity. Increasing quantities of water are required to return the available soil moisture to field capacity as the root zone expands. For example, 1.648, 1.957, 2.773, and 3.475 inches of water are applied for each of the first four irrigations, respectively.

The validity of simulation estimates is checked against the field observation. Table 16 shows the simulation estimates and actual observation of plot 29 of Colby, Kansas corn experiment. Both timing of irrigation and the quantity of water applied are interestingly close.

Table 16. Comparison of the results of simulation and actual corn grain experiments in Colby, Kansas, 1971

Simulation			Actual		
Date	Irrigated when ASM at (%)	Water applied (in.)	Date	Irrigated when ASM at (%)	Water applied (in.)
6-23-71	70	1.648			
6-30-71	70	1.957	6-21-71	77	2.75
7-18-71	70	2.773	7-20-71	79	2.54
7-31-71	70	3.475	7-30-71	71	3.53
8-13-71	70	3.352	8-13-71	66	4.19
8-26-71	70	3.610	8-26-71	65	4.27
9-10-71	70	3.525	9-10-71	66	4.17
Total		20.340			21.45

Limitations of Simulation Model

One of the limitations of simulation model is related to the inadequacy of available data which, in turn, required making the over-simplified assumptions that underlie the model. Where the various parameters are not known, values for other locations were taken and modified as proxies for conditions in western Kansas.

One of the most important limitations of the model is that crop growth rate at any stage is independent of crop growth at any other stage. As mentioned in Chapter 2, some studies (Hagan _et al._, 1957) indicated that there are functional relationships between plants' growth stages and their respective soil moisture levels. Such a function can be built to overcome these limitations.

In the model water application is not discounted for irrigation inefficiency. In reality, water applied in excess of that needed to bring the soil profile to field capacity is lost due to deep percolation and runoff, the application rate for each irrigation should be calculated as that needed to return the soil water to field capacity plus an allowance for inefficiency. Such limitations can be handled by making a small modification in the model.

The soil moisture is the only explanatory variable in the model, and no substitution between irrigation water and other inputs is considered. One other limitation of the model is that crop recovery is instantaneous after soil moisture stress conditions have been through irrigation.

Recognizing these important shortcomings of the simulation model, it nonetheless can provide empirically useful information. As an alternative means of predicting crop response to various irrigation strategies and weather conditions, simulation could assist the farmers in both allocating their limited supplies of water between competing crops and in the timing of irrigation to individual crops.

CHAPTER V. OPTIMAL TIMING OF IRRIGATION AND DYNAMIC PROGRAMMING

In the preceding chapter, it is assumed that there is sufficient irrigation water available to maintain the optimal crop growth. In many instances water is a limiting factor, and it is impossible to irrigate the crop as often as required in any stage. Under these circumstances the irrigator must determine how best to allocate a given quantity of water over crop's life span. Using the simulation model, it is possible to determine the optimal soil moisture policy for the various combinations of available water and soil moisture levels. This approach would involve hugh computational burdens.

In order to determine the optimal distribution of a given quantity of irrigation water over the season, the effect of different soil moisture strategies in different stages of plant growth on total return can be calculated by using the simulation-dynamic programming technique developed by Dudley (1969).

Instead of simulating sequentially through the stages in chronological order, the usual conventional dynamic programming will be used in which crop growth will be simulated beginning with the last stage. For each given combination of beginning water supply and the available soil moisture level, an optimal irrigation policy for first stage will be estimated. By using

Bellman's (1957) Principle of Optimality[1] and simulating crop growth over the second-last stage of the season, the optimal soil moisture policy which maximizes the return over the last two stages can be derived. By progressing sequentially backward through the irrigation season the optimal distribution of a given quantity of water over the entire season can be estimated.

Review of Literature

In recent years various attempts were made to solve the problem of timing of application and amounts of water to be applied each time by formulating it as a sequential decision process. Flinn and Musgrave (1967) and Flinn (1968) have shown that dynamic programming can be used to specify the optimal allocation of a given quantity of irrigation water over the irrigation season. Their analysis supported the view that the time of application of water is more important in determining its contribution to productivity than the total quantity applied.

Dudley (1969) built a stochastic dynamic programming model to overcome the three main limitations of the Flinn-Musgrave model. First, the Flinn-Musgrave model had only one state variable, namely, the quantity of water available for allocation over the remainder of the season. The implication of this was that the soil moisture

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[1]Principle of Optimality states that an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

level at the start of a stage would have no effect on the response to applied water during that stage. This implication is unrealistic. To overcome this limitation, Dudley's model possesses two state variables. They are the soil moisture level at the start of a stage and the available water supply.

Second, the Flinn-Musgrave model is a deterministic model. Their model determines the single-valued estimates of return in response to each quantity of irrigation water applied in each stage. According to Dudley, to examine the implications of irrigation planning in a variable environment, the dynamic programming model must be able to handle probability functions of return rather than single-valued estimates of expected return. As a result, Dudley developed a stochastic dynamic programming model to determine the optimal allocation of a given quantity of irrigation water over an irrigation season.

Third, in the Flinn-Musgrave model the decision variable was the number of irrigations, which is incompatible with the stochastic model used by Dudley. In Dudley's model terminal soil moisture is employed as the decision variable.

Hall and Butcher (1968) presented a deterministic dynamic programming model to estimate the optimum usage of irrigation water supplies, particularly in a season where there is insufficient water for all demands. The model had two state variables: (1) the soil moisture content and (2) the total amount of water available at the beginning of the season. The decision variable is the quantity of water transferred from the water supply to the soil through an irrigation process. A feature of Hall and Butcher's model was the multiplicative relationship between the sequential steps rather than the usual additive form. The equivalence between multiplicative and additive forms was achieved by making a logarithmic transformation of the multiplicative form.

In a review of the Hall and Butcher model, Aron (1969) among other criticisms pointed out that the sequential, recursive equations used to maximize the objective function which involved multiplicative production factor and additive cost factors violate Bellman's Principle of Optimality. The implication of this criticism was that irrigation costs do not affect the optimal policies. Later Hall and Dracup's (1971) formulation of dynamic programming ensured that irrigation costs do affect the optimal allocation of irrigation water over the season. A Joint Indian-American Team Report (1970) utilized the dynamic programming technique given by Hall and Dracup. The results confirmed the validity of their model for irrigating wheat.

Another stochastic dynamic programming approach was developed by de Lucia (1969). He estimated the relationship between stage of plant growth and variable soil moisture depletion levels so as to determine the optimal timing of irrigation. In his model, it is assumed that the contribution of each stage to the total yield will be linear rather than curvilinear as postulated by agronomists.

Burt and Stauber (1970) presented a two state variable stochastic dynamic programming model. The variables used to define the state of the system at the start of each stage were quantity of water in storage and a partial sum of the production function which measures the "crop condition". The decision variable in the Burt-Stauber model is net irrigation water applied where the stochastic nature of precipitation and other pertinent random weather inputs are incorporated into the objective function. They used the experimental results carried out in Missouri and a 74 year time series of precipitation and temperature data to determine the optimal allocation of limited irrigation water within the growing season of a single crop.

Asopa, Guise and Swanson (1973) used several models to estimate the best time of application of supplemental irrigation water in order to establish optimal operating policies in Illinois. They stated that among the various models, results from stochastic dynamic programming represent an improvement over comparable models and thus provide somewhat more realistic information.

Dynamic Programming Model

Bellman (1957) has helped to develop the theory of dynamic programming to facilitate the study of multistage decision processes. As shown in the Review of Literature, dynamic programming has proved to be a powerful technique for analyzing intertemporal distribution of irrigation water.

It is necessary to explain some basic concepts and definitions before developing the dynamic programming model. First, the decision process is divided into time periods or intervals called stages. In a multistage decision process, a sequence of decisions is sought which maximizes (or minimizes) some predefined objective function. In every process the decisions that are to be made relate to some system. The system is defined by the state variables which reflect the sum of all relevant information about the situation under consideration.

A series of decisions is made, one at each stage of the process. The effect of each decision is to determine a transformation of the state variables from their values at stage n to a new set of values at stage n + 1. In dynamic programming analysis, the state value at stage n + 1 depends on only the state at the previous stage n and the decision made at stage n.

Each stage of the total process yields a return (R). In the sort of process to which dynamic programming is applicable, the return from stage n may depend on the state at stage n, on the decision variable, and on time as represented by the stage number n.

Given the above definitions, the sequential decision process can be expressed in a functional equation form as:

$$f_N(X_N) = \text{Max} \quad R_N(X_N, P_N) + f_{N-1}(X_{N-1}) \qquad (39)$$

where

$f_N(X_N)$ = the total return from an N-stage process starting in state X where an optimal policy is used.

$R_N(X_N, P_N)$ = the return from the first stage of a process, starting in state X using decision P_N.

P_N = decision to be made when there are N stages left.

X_{N-1} = the new state resulting from decision P_N.

Equation (39) gives a recurrence relationship between the optimal return functions f_N and f_{N-1}. In order to solve this equation, it is necessary to know what the optimal return function is for some one value of N. The required initial solution can be found for a single-stage process, N = 1:

$$f_1(X_1) = \text{Max} \left\{ R_1(X_1, P_1) \right\} \tag{40}$$

The sequence of optimal return functions $f_1 \ldots f_N$ is given by equations (39) and (41). Solution of these functional recurrence equations is presented in various sources (Bellman, 1957; Dreyfus, 1961).

The model used in this chapter is developed by Dudley (1969). It is a combination of a soil moisture-plant growth simulation model and a dynamic programming model. First of all, for the Colby, Kansas area the irrigation season will be a 90-day period starting June 15 and ending September 15. This 90-day irrigation season was arbitrarily divided into six 15-day stages. The relationship between stages of decision processes and time periods is shown in

Figure 14. The state variables are defined as level of available soil moisture in the root zone and available water stock, both of which depend on weather conditions and irrigation treatments. To simplify the model, it is assumed that the addition to water stock during the irrigation season is zero. At each stage a set of relevant decisions or alternative courses of action exists concerning the quantity of water to apply, if any. This decision variable is called the terminal soil moisture level.

Generally, a decision at a specified stage and state of the process will change the state of the process in the subsequent stages. This is called state transition. Transition from one state to another can be stochastic or can be known with certainty. The model used here will be stochastic and will incorporate uncertain precipitation and other weather variables in terms of pan evaporation data.

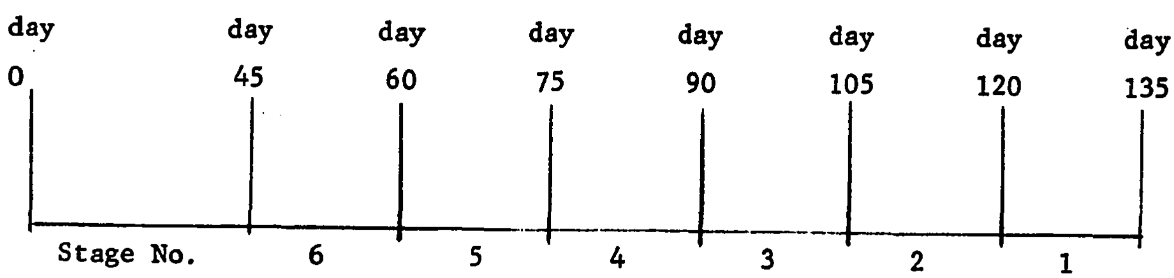

Figure 14. Dynamic programming stages and corresponding time periods

The objective of the model is to determine the optimal manner in which a given quantity of irrigation water can be distributed in each stage of season so as to maximize expected returns.

Given the above information Dudley's model can be written in a functional form as:

$$f_N(X_N,Y_N) = \text{Max} \quad E[r(X_N,Y_N,Z_C) + f_{N-1}(X_{N-1},Y_{N-1})] \qquad (41)$$

where

$f_N(X_N,Y_N)$ = the expected net returns from N-stage process under an optimal soil moisture policy when the initial state is that defined by X_N and Y_N;

X_N = available soil moisture percentage in the root zone at the start of stage N;

Y_N = the per acre water supply available for stage N;

Z_N = terminal soil moisture level maintained during stage N;

f_N = return from stage N;

E = mathematical expectation operator.

Maximization of equation (41) will determine the terminal soil moisture level which if maintained during stage N will result in maximizing the return from stage N plus the return from the (N-1) remaining stages given that an optimal policy will be followed during the remaining (N-1) stages.

As shown by Dudley, the problem can be reduced to a finite

Markovian decision process that can be solved on a digital computer by making the variables X, Y, and Z discrete, the latter explained by Burt (1964). Let X take on A discrete values X^1, X^2, ... X^a, ... X^A; let Y take on B discrete values Y^1, Y^2, ...Y^b, ... Y^B; and let Z in stage N take on C discrete values Z_{N1}, Z_{N2}, ... Z_{Nc}, ... Z_{NC}. X^a, Y^b, Z_c refer to the midpoints of the class interval. Since Z is a controlled variable, it is always set equal to a midpoint value while any X and Y values will be rounded to the nearest mid-point.

Let the probability of transition from state i at stage N to state j at stage N-1 be denoted by P_{ij}. Assume P_{ij} is independent of N. This set of transition probabilities can be represented by the transition matrix P:

$$P = \begin{pmatrix} P_{11} & \cdots\cdots & P_{1j} & \cdots\cdots & P_{1m} \\ P_{i1} & \cdots\cdots & P_{ij} & \cdots\cdots & P_{iM} \\ P_{m1} & \cdots\cdots & P_{Mj} & \cdots\cdots & P_{MM} \end{pmatrix} \qquad (42)$$

The M rows and columns correspond to the M feasible states of the system. The elements in row i are the probabilities of transition from state i to state j. Thus

$$0 \leq P_{ij} < 1 \qquad (43)$$

and

$$\sum_{j}^{M} P_{ij} = 1 \tag{44}$$

This transition matrix can be constructed for X and Y. Let pX_{Nc}^{ad} be the probability of available soil moisture changing from discrete level a to discrete level d over state N given a Z policy of c. The transition matrix pX_{nc} is

$$pX_n = \begin{pmatrix} p^{11} & \cdots\cdots & p^{1d} & \cdots\cdots & p^{1A} \\ p^{a1} & \cdots\cdots & p^{ad} & \cdots\cdots & p^{aA} \\ p^{A1} & \cdots\cdots & p^{Ad} & \cdots\cdots & p^{AA} \end{pmatrix} \tag{45}$$

where the A rows and columns correspond to the A feasible available soil moisture states of the system. The elements in row a are the probabilities of transition from state a to state d, given a Z policy of c in stage N with a Y of e. Thus

$$0 \leq p^{ad} \leq 1 \tag{46}$$

and

$$\sum_{d=1}^{A} p^{ad} = 1 \tag{47}$$

Since there are N stages, C Z policies and B water supply levels, there are NCB of these transition matrices.

Let pY_{nc}^{be} be the probability of the water supply changing from discrete level b to discrete level e during stage N given a terminal soil moisture policy of c. The transition matrix pY_{nc} can be formed as

$$pY_{nc} = \begin{pmatrix} p^{11} & \cdots\cdots & p^{1e} & \cdots\cdots & p^{1B} \\ p^{b1} & \cdots\cdots & p^{be} & \cdots\cdots & p^{bB} \\ p^{B1} & \cdots\cdots & p^{Be} & \cdots\cdots & p^{BB} \end{pmatrix} \tag{48}$$

The elements in row b are the probabilities of transition from state b to state e, given a beginning-stage soil moisture level of a, and maintaining a terminal soil moisture policy of c in stage N. Thus

$$0 \leq p^{be} \leq 1 \tag{49}$$

and

$$\sum_{e=1}^{B} p^{be} = 1 \tag{50}$$

There are NCA of these matrices.

Corresponding to the transition matrix is a return matrix also formed by Dudley. Let $r_N^d\ (Y_N^b,\ X_N^a,\ Z_c\ )$ refer to the net return resulting from maintaining a terminal soil moisture policy of c in stage N in which soil moisture changes from a to d given a beginning stage water supply of b. The return matrix r_N^d is formed as

$$r_N^d \; (Y_N^b, \; X_N^a, \; Z_c) = \begin{pmatrix} r^{11} \; \ldots\ldots \; r^{1d} \; \ldots\ldots \; r^{1A} \\ r^{a1} \; \ldots\ldots \; r^{ad} \; \ldots\ldots \; r^{aA} \\ r^{A1} \; \ldots\ldots \; r^{Ad} \; \ldots\ldots \; r^{AA} \end{pmatrix} \tag{51}$$

The element r^{ad} represents the return from a transition from state X^a to state X^d, given a beginning-stage water supply of b and by maintaining a terminal soil moisture level of c in stage N. Return estimates are independent of the change in water supply during the stage but are a function of the transiton in soil moisture.

As shown by Dudley, these rewards are not only a function of the magnitude of the difference between the beginning and ending stage available soil moisture levels, but also depend on the pattern of soil moisture variation during the stage, and evapotranspiration during the stage, and the cost of irrigation during the stage.

The means of these estimates are chosen as the parameters to use in the dynamic programming formulation. For example, the element r^{ad} in the reward matrix is the expected return resulting from a change in available soil moisture from discrete level d during stage N, given a terminal soil moisture policy of c with the beginning water supply of b. The term expected return is used for estimates of return resulting from maintaining a given terminal soil moisture level for a given water supply and available soil moisture during the stage. The expected return associated with an optimal terminal soil moisture policy for a one stage process is given by

$$f_1(X_1^a,\ Y_1^b) = \underset{c}{\mathrm{Max}}\left\{\sum_{d=1}^{A} pX_{1c}^{ad}\ r_1^d(X_1^a,\ Y_1^b,\ Z_c)\right\} \tag{52}$$

Since $\sum_{e=1}^{B} pY_{Nz}^{be} = 1$,

equation (52) may be written as

$$f_1(X_1^a,\ Y_1^b) = \underset{c}{\mathrm{Max}}\left\{\sum_{d=1}^{A}\ \sum_{e=1}^{B} pX_{1c}^{ad}\ pY_{1c}^{be}\ r_1^d(X_1^a,\ Y_1^b,\ Z_c)\right\} \tag{53}$$

For a n-stage process, the recursive relationship is:

$$f_N(X_N^a, Y_N^b) = \underset{c}{\mathrm{Max}}\left\{\sum_{d=1}^{A}\ \sum_{e=1}^{B} pX_{Nc}^{ad}\cdot pY_{Nc}^{be}\cdot[Nr_N^d\ (X_N^a, Y_N^b, Z_c) + f_{N-1}(X_{N-1}^a, Y_{N-1}^b)]\right\} \tag{54}$$

In order to solve equation (53) for optimal return function it is necessary to find the optimal terminal soil moisture level (Z) for stage 1. Solution of equation (52) will determine the optimal Z for stage 1. The results from the first stage process are then used to find the optimal Z policy for a two-stage process while, in turn, is used in the subsequent process until the optimum is obtained in a n-stage process.

The computer program is written by Dudley (1969). With modified form it is presented in Appendix C.

Results from Dynamic Programming Model

Given the 90-day irrigation season which is divided into six 15-day stages, crop growth will be simulated starting from the last stage of the season. Recall that for each given combination of available soil moisture and water supply, the optimal water application policy will be determined for that stage. The results of the last stage will be used to determine the optimal irrigation strategy in the last two stages which, in turn, are used in subsequent periods until the optimum is obtained for a six stage process.

Twenty years of daily pan evaporation and precipitation data were obtained from the Colby Experiment Station in Kansas. Given this information and relevant combinations of beginning soil moisture and available water supply, daily growth was simulated for each stage. Each stage was replicated 20 times to reflect the twenty years of weather data. Transition matrices whose elements represent the probability of transition from state i at stage N to state j at stage N-1 and the corresponding reward matrix for any specific stage are estimated. As noted earlier, the two state variables are available soil moisture in the root zone and the per acre water supply available for irrigation. Given these matrices, the dynamic programming model will determine the optimal irrigation strategy and corresponding expected net returns.

Available soil moisture in the beginning stage (X) was divided in the following 6 discrete intervals: 0, 1-20, 21-40, 41-60, 61-80, 81-100 percent as represented by the midpoints $X_N^1 = 0\%$, $X_N^2 = 10\%$, $X_N^3 = 30\%$, $X_N^4 = 50\%$, $X_N^5 = 70\%$, $X_N^6 = 90\%$ respectively for the Nth stage. The per acre water supply available in the beginning stage (Y) was divided into 9 discrete units of 3 inches each. That is, $Y_N^1 = 0$, $Y_N^2 = 3$, $Y_N^3 = 6$, $Y_N^4 = 9$, $Y_N^5 = 12$, $Y_N^6 = 15$, $Y_N^7 = 18$, $Y_N^8 = 21$, $Y_N^9 = 24$ inches. Terminal soil moisture (Z) in each stage was divided into 7 discrete intervals as 0, 1-20, 21-40, 41-60, 61-80, 81-100 percent and represented by midpoints $Z_{N1} = 0\%$, $Z_{N2} = 10\%$, $Z_{N3} = 30\%$, $Z_{N4} = 50\%$, $Z_{N5} = 70\%$, $Z_{N6} = 90\%$ for stage N. Z_{N6} is changed from 90% to 80% because it is assumed that no extra growth would be obtained from the higher level terminal soil moisture policy.

For a given combination of the beginning stage soil moisture and water supply, the optimal terminal soil moisture and corresponding expected net return are represented in Table 17. For example, the optimal policy in stage 3 for a beginning soil moisture level of 50 percent and a beginning water supply of 9 inches is 80 percent and the expected net return from the optimal policy maintained over the remainder of the season is \$54.54.

Some of the important features of the computer results presented in Table 17 will be analyzed in the following discussion.

Consider the last growth stage, i.e., stage 1. With an exception of one column, all terminal soil moisture values are zero and corresponding expected return values are non-negative. The implication of this is that no irrigation water is necessary and

Table 17. Computer output results of dynamic programming

Y[b]	X=0[a] Z[c]	E(R)[d]	X=10 Z	E(R)	X=30 Z	E(R)
			Stage 1			
0	0.0	0.0	0.0	1.26	0.0	1.21
3	0.0	0.0	0.0	1.26	0.0	1.21
6	0.0	0.0	0.0	1.26	0.0	1.21
9	0.0	0.0	0.0	1.26	0.0	1.21
12	0.0	0.0	0.0	1.26	0.0	1.21
15	0.0	0.0	0.0	1.26	0.0	1.21
18	0.0	0.0	0.0	1.26	0.0	1.21
21	0.0	0.0	0.0	1.26	0.0	1.21
24	0.0	0.0	0.0	1.26	0.0	1.21
			Stage 2			
0	0.0	0.0	0.0	5.45	0.0	5.35
3	0.0	0.0	0.0	5.45	0.0	5.35
6	0.0	0.0	0.0	5.45	0.0	5.35
9	0.0	0.0	0.0	5.45	50.0	16.73
12	0.0	0.0	30.0	15.27	80.0	17.45
15	0.0	0.0	80.0	16.00	80.0	17.45
18	0.0	0.0	80.0	16.00	80.0	17.45
21	0.0	0.0	80.0	16.00	80.0	17.45
24	0.0	0.0	80.0	16.00	80.0	17.45
			Stage 3			
0	0.0	0.0	0.0	15.19	0.0	14.97
3	0.0	0.0	0.0	15.19	0.0	14.97
6	0.0	0.0	0.0	15.19	0.0	14.97
9	0.0	0.0	30.0	15.94	30.0	38.84
12	0.0	0.0	10.0	36.89	80.0	54.94
15	0.0	0.0	80.0	53.48	80.0	60.53
18	0.0	0.0	80.0	59.07	80.0	60.50
21	0.0	0.0	80.0	59.04	80.0	60.41
24	0.0	0.0	80.0	58.95	80.0	60.41

[a]Available soil moisture percentage in the root zone at the start of stage N.

[b]The per acre available water supply for stage N.

[c]Terminal soil moisture level maintained during stage N.

[d]Expected economic return.

X=50		X=70		X=90	
Z	E(R)	Z	E(R)	Z	E(R)
0.0	1.11	0.0	1.10	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	1.11	80.0	1.20	0.0	3.73
0.0	5.22	0.0	4.73	0.0	18.52
0.0	5.22	0.0	4.73	80.0	25.97
70.0	6.48	80.0	20.27	80.0	25.85
80.0	18.91	80.0	20.37	80.0	25.85
80.0	18.91	80.0	20.37	80.0	25.85
80.0	18.91	80.0	20.37	80.0	25.85
80.0	18.91	80.0	20.37	80.0	25.85
80.0	18.91	80.0	20.37	80.0	25.85
80.0	18.91	80.0	20.37	80.0	25.85
0.0	14.34	0.0	13.01	0.0	27.03
0.0	14.34	80.0	15.40	80.0	55.85
70.0	21.39	80.0	53.79	80.0	64.84
80.0	54.54	80.0	62.33	80.0	68.19
80.0	61.99	80.0	63.42	80.0	68.12
80.0	61.96	80.0	63.33	80.0	68.07
80.0	61.87	80.0	63.33	80.0	68.07
80.0	61.87	80.0	63.33	80.0	68.07
80.0	61.87	80.0	63.33	80.0	68.07

Table 17. (continued)

Y	X=0 Z	X=0 E(R)	X=10 Z	X=10 E(R)	X=30 Z	X=30 E(R)
			Stage 4			
0	0.0	0.0	0.0	23.21	0.0	22.86
3	0.0	0.0	0.0	23.21	0.0	22.86
6	0.0	0.0	0.0	23.21	0.0	22.86
9	0.0	0.0	30.0	25.87	30.0	40.41
12	0.0	0.0	80.0	54.92	80.0	58.94
15	0.0	0.0	80.0	82.79	80.0	84.06
18	0.0	0.0	80.0	85.12	80.0	94.03
21	0.0	0.0	80.0	92.76	80.0	94.82
24	0.0	0.0	80.0	93.55	80.0	97.67
			Stage 5			
0	0.0	0.0	0.0	33.30	0.0	32.29
3	0.0	0.0	0.0	33.30	0.0	32.29
6	0.0	0.0	50.0	37.83	50.0	52.65
9	0.0	0.0	80.0	57.04	80.0	69.11
12	0.0	0.0	80.0	75.05	80.0	89.79
15	0.0	0.0	80.0	88.83	80.0	91.79
18	0.0	0.0	80.0	106.85	80.0	109.82
21	0.0	0.0	30.0	115.15	80.0	125.39
24	0.0	0.0	80.0	126.16	80.0	127.52
			Stage 6			
0	0.0	0.0	0.0	45.30	0.0	44.82
3	0.0	0.0	80.0	50.45	80.0	52.85
6	0.0	0.0	80.0	69.87	80.0	76.75
9	0.0	0.0	30.0	89.43	80.0	95.47
12	0.0	0.0	80.0	97.94	50.0	107.02
15	0.0	0.0	80.0	111.38	80.0	114.39
18	0.0	0.0	30.0	123.52	50.0	124.17
21	0.0	0.0	80.0	128.09	80.0	128.74
24	0.0	0.0	70.0	136.86	80.0	144.92

X=50		X=70		X=90	
Z	E(R)	Z	E(R)	Z	E(R)
0.0	22.26	0.0	20.60	0.0	32.52
0.0	22.26	80.0	23.39	80.0	25.49
50.0	41.58	80.0	58.74	80.0	75.52
80.0	60.22	80.0	86.60	80.0	92.82
80.0	85.33	80.0	88.94	80.0	96.86
80.0	87.67	80.0	96.58	80.0	102.14
80.0	95.75	80.0	97.36	80.0	105.16
80.0	98.74	80.0	100.38	80.0	105.13
80.0	99.10	80.0	100.31	80.0	105.09
0.0	31.18	0.0	31.06	0.0	45.74
70.0	34.70	80.0	55.33	70.0	66.54
80.0	66.85	80.0	72.41	70.0	82.35
80.0	88.45	80.0	91.93	80.0	96.69
80.0	92.75	80.0	94.94	80.0	115.73
80.0	108.78	80.0	112.84	80.0	120.91
50.0	113.96	80.0	127.53	80.0	133.22
80.0	128.08	80.0	129.87	80.0	134.43
80.0	131.71	80.0	133.31	80.0	137.33
0.0	45.46	0.0	46.02	0.0	58.80
70.0	69.26	80.0	77.50	80.0	83.12
70.0	89.71	80.0	95.72	80.0	103.93
80.0	99.23	80.0	99.88	80.0	103.93
80.0	111.09	80.0	114.10	80.0	119.73
80.0	115.04	70.0	121.90	80.0	127.62
80.0	127.95	80.0	130.03	80.0	134.07
70.0	138.16	80.0	145.26	80.0	153.11
80.0	148.42	80.0	149.06	80.0	153.11

rainfall is sufficient to maintain the potential growth.

In general, the optimizing level of terminal soil moisture policy is 80 percent. This means that the farmer should irrigate whenever the available soil moisture in the root zone falls to 80 percent of field capacity, assuming that he has sufficient water.

The expected economic return to increasing beginning stage soil moisture levels (X) when the beginning stage water supply (Y) is zero is always positive. The implication of this is that corn can be produced on dry land assuming the available soil moisture is at that level specified by the value of X.

The expected economic return responses to increasing the beginning stage water supply when the beginning soil moisture levels are zero are always zero. This is an expected result because an assumption that when beginning soil moisture is zero, it is assumed that the optimal terminal soil moisture and expected return are automatically zero is built into the model. In other words, crop death occurs in the beginning stage, and the expected return values would be zero in that and subsequent stages.

The expected economic return responses to an increase in beginning soil moisture levels in each stage given a beginning supply of 12 inches are plotted in Figure 15. Recall that stage 1 of the irrigation period represents the stage where the plant is approaching maturity. Consequently, the expected-return response for stage 1 in Figure 15 is essentially horizontal. That is, as the plant nears maturity, the level of beginning soil

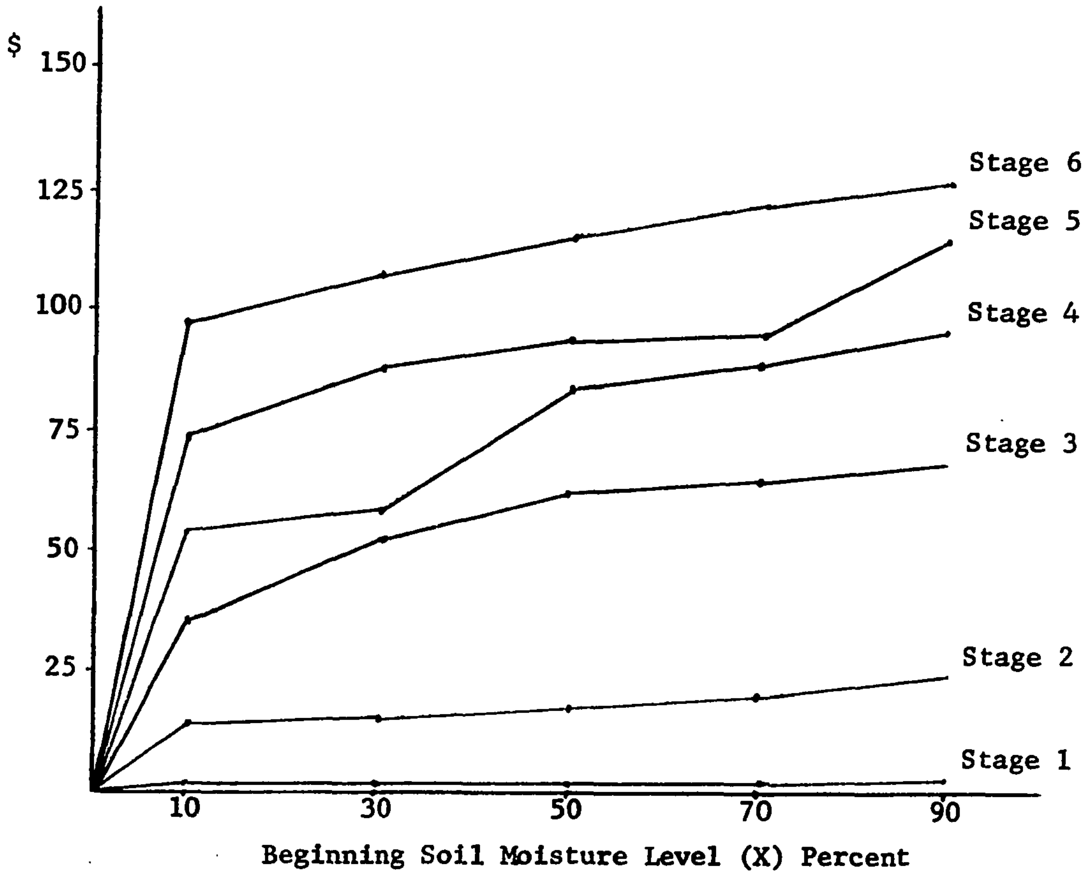

Figure 15. Expected economic return in response to increases in beginning soil moisture levels in each stage given a beginning water supply of 12 inches

moisture for stage 1 has little impact on expected returns. In contrast, stage 6 represents the first 15-day of the irrigation season. The expected-return curve for stage 6 is considerably above that for the other stages, especially at lower beginning soil moisture levels. When the initial soil moisture level is in excess of 50 percent, the expected return curves for stages 4, 5, and 6 appear to tend to converge. The curves flatten out at higher initial soil moisture levels partly because of the decreasing cost of irrigation associated with higher soil moisture content. Less water will be applied to bring the soil to field capacity. Further, as beginning soil moisture increases it is likely that less crop stress occurs which, in turn, means that actual yield will be closer to the potential yield.

The expected economic return to increasing levels of beginning water supply in each stage given a constant beginning soil moisture of 50 percent are plotted in Figure 16. The horizontal economic response curve for a water supply greater than 12 inches for stages 1, 2 and 3 indicate an excess of available water in these stages. The slopes of the curves in Figure 16 can be called the marginal expected return, i.e., the addition to total expected return from a one unit increase in the water supply. The slopes are much higher for all specified water supply levels for the earlier stages such as stages 4, 5 and 6. For these stages, marginal expected return is increasing even at relatively high levels of water supply. In later stages, i.e., 1, 2 and 3, marginal expected returns are relatively small in response to increasing water supplies even at lower supply levels. This is because small quantities of water are insufficient to bring the root zone to field capacity when the beginning soil moisture is 50 percent. In earlier

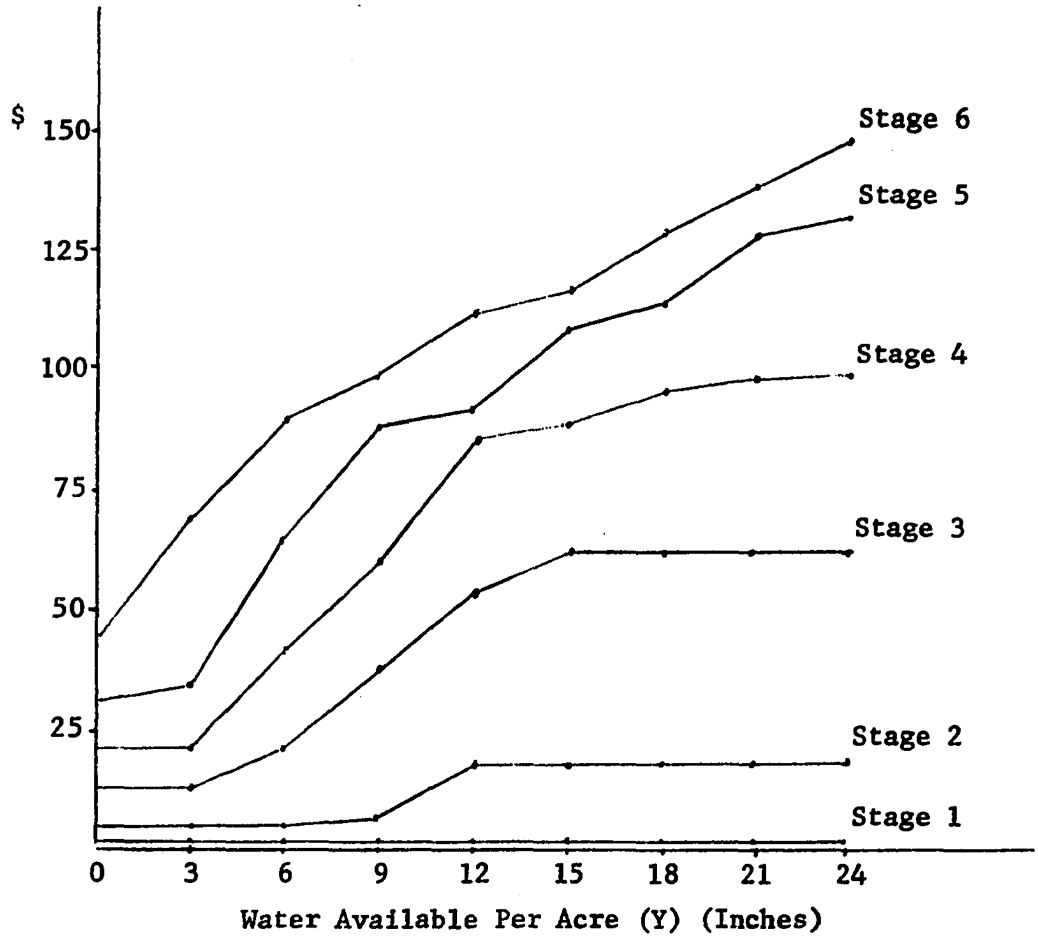

Figure 16. The expected economic return in response to various levels of beginning water supply in each stage, given a constant beginning soil moisture of 50 percent

stages, i.e., 4, 5 and 6, the marginal expected return is increasing in response to lower water supply levels because the root zone is still relatively small and less water is required to bring the soil to field capacity.

In summary the primary objective of this chapter was to incorporate the crop growth simulation model into a dynamic programming model and to determine the optimal allocation of a given quantity of water over an irrigation season, in Colby, Kansas. Limitations of the crop growth simulation model are also limitations of the dynamic programming model, Dudley (1969). As improved simulation models are constructed for predicting the crop response to water increases, it is expected that the results of dynamic programming models such as the above would be close to optimal.

CHAPTER VI. SUMMARY AND CONCLUSIONS

This is an era of rapid change in agriculture. Changes are occurring in both the physical production setting and the economic environment in which farmers must make decisions. Proper resource management requires a good understanding of the response of crops to the application of various production inputs. Among these inputs irrigation water is an important factor of production in many regions. Used in combination with other inputs such as fertilizer, water may increase dry land crop yields substantially in arid or semi-arid areas. If crop response to water could be estimated with reasonable accuracy, farmers could increase net return by making more timely irrigation and/or perhaps by applying less water.

The objective of this study has been to present and apply alternative optimization models for determining the efficient use of water as applied to an individual crop. In Chapter 2, an overall picture of the soil-water-plant system was reviewed for those readers not having a soil science background. The relationship between a plant and the variables affecting its growth is extremely complex. Regarding the plant-water relationship, it would appear that plant growth is not only a function of the soil moisture level but also depends on those factors associated with the plant's water balance. The latter, in turn, depends on the relative rates of water absorption and water loss. It is concluded that irrigation should be considered when water loss is greater than absorption at some predetermined soil moisture level.

In Chapter 3, a production function for water was estimated by abstracting from detailed relationships involved in growth processes and concentrating on the general relation of corn grain yields to water applications. The model utilizes conventional production function analysis to derive the yield response function where water is the independent variable. The criticism of this approach is that it fails to consider the timing of water applications. To overcome this limitation the distribution of water is implicitly incorporated into the experimental design used in the field study.

The data were generated from fertilizer and irrigation trials in 1971 at the Colby Branch Station, Kansas State University. Several alternative functional forms were considered as a basis for estimating the yield response of corn to nitrogen fertilizer and irrigation water. The quadratic function was chosen for more intensive physical and economic analyses. Statistical analysis indicated that about 94 percent of the variation in observed yields was explained by the applied water and nitrogen fertilizer inputs.

The response surface for the quadratic function and corresponding marginal products, isoquants and isoclines were estimated and analyzed. Given the estimated production function and the specified price relationship, the optimum rates of water and nitrogen fertilizer were derived. Under certainty and unlimited capital, the profit-maximizing quantities of water and fertilizer were derived as 23.35 acre inches of water and 272 pounds of nitrogen fertilizer. In comparison, the yield-maximizing water and fertilizer levels were estimated to be 24.9 acre inches of water and 295 pounds of nitrogen fertilizer. These are similar to the profit-maximizing input levels. In other words, for given prices of water, nitrogen fertilizer and corn as $0.60/acre-inch of water, $0.08/pound of fertilizer, and $0.024/pound of corn, maximizing yields and maximizing profit results in nearly the same economic returns.

The crop production function was utilized to derive a static-normative demand function for water representing the quantities of water which should be purchased at corresponding water prices. For both the short-run and long-run demand functions, the price elasticity of demand for water is quite low.

In addition to the conventional production function analysis, linear programming was used to determine the optimal allocation of water and other limited resources at the farm level. The previously estimated production function was used to derive per acre water requirement for corn production activities. These activities were incorporated in the linear programming model. The optimal solution for the model shows that corn will be produced with 18 acre-inches of water instead of 23.4 acre inches water as estimated in the production function analyses.

In Chapter 4, a soil moisture-plant growth simulation model was used to estimate the timing and the amount of irrigation water needed to provide adequate soil moisture for realizing optimum yield levels. Yield reductions from alternative soil moisture stress conditions were also estimated. The model incorporated the important effects of soil and climatic variables. Soil was represented by the water-holding capacity of soil and climate was represented by pan evaporation rates and rainfall.

Some of the variables required for the simulation model were obtained from Colby Experiment Station, Kansas. In those cases were specific values of some variables were not available, values for other locations were taken and modified as a proxies for conditions in western Kansas.

In the computerized simulation model used in this study, the criterion used was to apply irrigation water whenever the existing available soil moisture in the root zone fell below 70 percent of field capacity, taking into account the predicted rainfall and evapotranspiration which would occur on that day. Corn was subject to stress in 29 days out of the 135-day growing season. If corn were irrigated whenever the soil moisture level fell to 80 percent of field capacity, only five days of stress would occur. The total water applied for irrigation as derived in the simulation model was comparable to that estimated by the production function analyses. The difference between the two approaches is about 3 acre-inches.

In both the production function and simulation models it is assumed that response to water in each growth stage is independent of response to water in all other stages. It is further assumed the quantity of water is unlimited. But, as noted in Chapter 2, an adverse affect of soil moisture on the growth rate at any particular stage may affect subsequent growth and total yield. Further in most instances water is not available in unlimited amounts.

In Chapter 5, a dynamic programming model was used to determine the optimal distribution of a given quantity of water over the growing season. In the model the effect of different soil moisture strategies in different stages of plant growth on total economic returns was estimated. The dynamic programming model used is a stochastic model because it incorporates varying rainfall conditions and water requirements of the crop.

Some assumptions had to be made in order to apply the dynamic programming model to the conditions existing at Colby, Kansas. A 90-day irrigation season starting June 15 and ending September 15 was assumed. This 90-day irrigation season was arbitrarily divided into six 15-day stages. In order to estimate yield response under varying climatic conditions, twenty years of precipitation and pan evaporation data were incorporated into the dynamic programming model.

The most important result generated by the dynamic programming model is that a decision maker should irrigate whenever the available soil moisture in the root zone fell to 80% of field capacity, assuming that enough water was available to bring the whole root zone to the field capacity level. Dynamic programming, simulation and production function analysis generated similar water-use levels for producing profit maximizing yields. In each case, the irrigation decision rule is to irrigate corn when soil moisture in the root zone (top two feet in production function analysis) is depleted to the 80 percent of field capacity level.

As noted throughout the study, the results of each model are limited on both agronomic and economic grounds. Most of these limitations are related to the inadequacy of available data which, in turn, require making the oversimplified assumptions that underlie each model. Most of the parameters used in the simulation and dynamic programming models can only be regarded as approximate. If the various model components were more accurately determined, it is expected that an operationally-useful simulation model would allow better prediction of crop yields in relation to water applied or any other input. Until an extended simulation model is developed, field experiments over a period of several years would be necessary in order to generate necessary empirical data. One of the limitations of conventional production function analysis is that crop responses to water among stages are not known. Thus, field experiments should be directed toward estimating the crop response to water and different stages and also the interaction among stages.

In general, the potential use of optimization models in solving practical problems is dependent upon more detailed quantitative formulations of crop-water relationships. These formulations must be based on suitable experimental data. This requires continuing cooperation and coordination among agronomists, engineers, economists, and professionals from other related fields.

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ACKNOWLEDGMENTS

I would like to express my appreciation to several individuals who have contributed to the development of this study.

I would first like to thank Dr. Earl O. Heady, my major professor, for suggestions and constructive criticism at different times throughout the writing of this thesis. Thanks is also due to his advice and encouragement throughout my graduate program.

A sincere thanks is expressed to Dr. Robert H. Shaw and Dr. Vince A. Sposito for their time and valuable assistance.

I would also like to thank Dr. Roger Hexem for reading the thesis and making constructive comments.

Last, but not least, my deepest appreciation and gratitude are extended to Nurcan, Selcuk, and Timur who are my closest friends in times of joy and sorrow.

APPENDIX A. THE DATA USED IN FORMING THE PRODUCTION FUNCTION EQUATIONS (11), (12) AND (13)

Yield (lb./acre)	Treatment: ASM (%)	Water applied (in./acre)	Nitrogen applied (lb./acre)
1873	20	9.60	0
1711	20	8.32	0
956	20	8.38	0
1600	20	8.85	0
3021	50	11.66	0
2992	50	10.61	0
1293	50	8.31	0
1276	50	9.51	0
2708	80	18.23	0
1560	80	17.67	0
1421	80	14.43	0
2969	80	21.52	0
6518	35	8.16	90
5758	35	10.08	90
7214	65	27.56	90
6802	65	18.00	90
5984	20	9.56	180
6170	20	10.13	180
7144	20	10.19	180
6164	20	9.01	180
8205	50	15.37	180
8605	50	16.61	180
8605	50	13.36	180
8651	50	15.92	180
9266	80	22.07	180
9232	80	24.67	180
8321	80	22.30	180
8826	80	22.97	180
6790	35	10.08	270
7335	35	8.87	270
10154	65	25.80	270
7805	65	23.80	270
6309	20	9.12	360
6448	20	10.32	360
6593	20	10.28	360
6680	20	10.77	360
9371	50	14.58	360
8663	50	14.49	360
8200	50	13.92	360
8275	50	12.84	360
10409	80	23.04	360
9568	80	26.48	360
9667	80	21.45	360
8646	80	25.90	360

APPENDIX B. COMPUTER PROGRAM FOR CROP-GROWTH SIMULATION MODEL

```
C THIS PROGRAM HAS BEEN ADAPTED AND MODIFIED FROM THE
C ORIGINAL CODING OF DUDLEY (1969).
C
C VARIABLES USED:
C EV             DAILY PAN EVAP VECTOR, IN.
C RF             DAILY RAINFALL VECTOR, IN.
C F              CROP FACTOR
C VM             MONETARY VALUE OF A DAY'S GROWTH OF
C                CROP ($)
C EAT            DAILY ACTUAL EVAPOTRANSPIRATION, IN.
C ETT            DAILY POTENTIAL EVAPOTRANSPIRATION, IN.
C CI             VARIABLE COST OF IRRIGATION ($)
C COSTWT         FIXED COST OF IRRIGATION ($)
C SM             AVAILABLE SOIL MOISTURE ON DAY K,IN.
C HSMA           AVAILABLE SOIL MOISTURE ON DAY K-1,IN.
C WS             WATER SUPPLY PER ACRE,IN.
C RBT            RAIN BALANCE,IN.
C  DEFINE THE FORMAT AND DIMENSION STATEMENTS
      DIMENSION EV(150),RF(150),TT(10,2),F(150),VM(150),
     1WT(150)
    1 FORMAT(9X,F5.2)
  471 FORMAT(15F4.3)
   45 FORMAT(1X,I3,11F10.3)
 9471 FORMAT(I4)
  600 FORMAT(15F4.2)
 5591 FORMAT(2F8.3)
  557 FORMAT(15F5.2)
  103 FORMAT(14H FNR,GRTH,SIG=(3(1X,E10.4)))
  168 FORMAT(33H SEV,SETT,SEAT,SRF,SEAAIG,SRFAIG=(6(E10.4)))
C  READ MODIFY AND STORE DATA
  555 READ 1,TT
      PRINT 4444,((TT(I,J),J=1,2),I=1,10)
 4444 FORMAT(10(/,2F10.2))
      READ 557,F
      READ 557,VM
      READ 5591,CI,COSTWT
      READ 9471,NYEAR
      YEARN=NYEAR
      LS=1
      DO 9472 J=1,NYEAR
      READ 471,EV
      READ 600,RF
      READ 557,WT
      DO 9482 K=1,150
C  ALL RAINFALL LESS THAN .10 IN. IS IGNORED
 9480 IF(RF(K)-.10)9481,9473,9473
 9481 RF(K)=0.
 9473 RF(K)=RF(K)
 9482 CONTINUE
```

```
 9472 CONTINUE
  667 LSHOLD=LS
 1235 LS=1
      KYEAR=0
  303 IF(KYEAR-NYEAR)450,48,48
  450 CONTINUE
      KYEAR=KYEAR+1
C  INITIALIZE THE VARIABLES
C  SETTING THE SOIL MOISTURE LEVEL (SM) TO FIELD CAPACITY (%)
      SM=100.
      BK=0.
      TSM=0.
      WS=40.
      HSMA=SM
      SMA=SM
      RBT=0.
      GSUM=0.
C  AVAILABLE SOIL MOISTURE IN THE TOP SIX IN.(CAPA)
      CAPA=1.220
      DEAD=0.
      SIG=0.
      SSIG=0.
      WTNG=0.
      GRTH=0.
      WTCOST=0.
      SGRTH=0.
      WTNGS=0.
      SWTCOS=0.
C  FIELD CAPACITY OF 6 FEET SOIL (CAPAMAX)
      CAPAMX=12.15
C  THE NUMBER OF DAYS DURING WHICH TIME EVAPOTRASPIRATION
C  IS ASSUMED TO BE SUPPLIED FROM GRAVITATIONAL WATER ONLY
      GRAVDY=1.
      DAY=0.
      KTN=0.
      SUMEV=0.
      SUMETT=0.
      SUMEAT=0.
      SUMRF=0.
      SRFAIG=0.
      SEAAIG=0.
C  INITIALIZE DAILY VARIABLES AND INCREMENT DAY COUNTER
    4 K=K+1
      DAY=DAY+1.
      VMK=VM(K)
C  TEST WHETHER OR NOT THE CROP HAS CEASED TRANSPIRING
C  DUE TO EITHER MATURITY OR DEATH
      IF(DEAD)251,251,250
  250 ETT=.09*EV(K)
```

```
      F(K)=.09
      PT=1.
      HSMA=HSMA*CAPA*.01*(100./12.15)
      CAPA=12.15
      TSM=-9.918
      VMK=0.
      RBT=0.
      WTN=0.
      GO TO 26
C  CALCULATION OF THE POTENTIAL EVAPOTRANSPIRATION
  251 PT=0.
      WTN=0.
      ETT=F(K)*EV(K)
C  DETERMINATION OF THE "RAIN BALANCE"
      IF(CAPA-1.59)770,771,771
C  RBTMX IS THE MAXIMUM QUANTITY OF ADDITIONAL WATER WHICH
C  CAN BE STORED IN THE UPPER 6 IN. OF SOIL
  770 RBTMX=CAPA-(CAPA*HSMA*.01)
      GO TO 772
  771 RBTMX=1.59-.0159*HSMA
  772 IF(RF(K)+RBT-RBTMX)721,721,720
  720 RBT=RBTMX-ETT
      GO TO 207
  721 RBT=RBT+RF(K)-ETT
  207 IF(RF(K))92,92,98
   92 IF(RBT)97,97,98
   97 RBT=0.
      GO TO 220
C  CALCULATION OF THE VALUE OF PT
   98 PT=1.
      IF(RBT)93,93,220
   93 RBT=0.
C  EXTENSION OF THE ROOT ZOON IF IT IS NOT ALREADY REACHED
C  ITS MAXIMUM
  220 IF(CAPAMX-CAPA)221,221,781
  781 IF(K-22)221,221,209
  209 HSMA=(((CAPA*HSMA*.01)+.1735)/(CAPA+.1735))*100.
      CAPA=CAPA+.1735
C  SELECTION OF THE APPROPRIATE PT VALUES FROM TABLE
  221 IF(PT)7,7,26
    7 AE=HSMA/10.
      KAE=AE
      FKAE=KAE
      K1=KAE+1
      IF(FKAE-AE)25,10,25
   10 K1=K1-1
   25 K1=10-K1+1
      IF(K1-10)8,8,9
    9 K1=10
```

```
      IF(K1.LE.0)K1=1
    8 K2=1
      IF(EV(K).GT..30)K2=2
      PT=TT(K1,K2)
C  CALCULATION OF THE ACTUAL EVAPOTRANSPIRATION
   26 EAT=PT*ETT
C  CALCULATION OF WHETHER OR NOT THE SOIL MOISTURE ON DAY K-1
C  PLUS THE RAINFALL ON DAY K LESS THAN THE TERMINAL SOIL
C  MOISTURE (TSM)
  210 IF(HSMA+(RF(K)*100./CAPA)-(EAT*100./CAPA)-TSM)58,58,216
C  CALCULATION OF THE QUANTITY OF WATER WHICH MUST BE APPLIED
   58 WTN=(100.-HSMA)*CAPA*.01
      IF(WS-WTN)217,59,59
   59 IF(SIG)725,780,725
  780 COSWT=0.
      GO TO 598
  725 COSWT=COSTWT
  598 WS=WS-WTN
      SIG=SIG*WTN
      WTCOST=WTCOST+COSTWT
      WTNG=WTNG+1.
      SMA=((HSMA*CAPA*.01)+WTN+RF(K))*100./CAPA
      GO TO 41
  217 WTN=0.
  216 SMA=((HSMA*CAPA*.010)+RF(K)-EAT)*100./CAPA
   41 IF(SMA-99.9)51,218,218
  218 SMA=100.
      DAY=0.
      RBT=0.
   51 IF(DAY-GRAVDY)699,699,698
  699 SMA=100.
  698 HSMA=SMA
      IF(DEAD)50,50,161
   50 IF(SMA)60,60,61
C  DETERMINE THE VALUE OF CROP GROWTH ON DAY K
   61 IF(PT-1.)54,62,55
   54 GI=1.-WT(K)
      GO TO 63
   62 GI=1.
      GO TO 63
   55 GI=0.
   63 GR=GI*VMK
C  SUM OF THE GROSS VALUE OF CROP GROWTH
      GSUM=GSUM+GR
      FNR=GSUM-SIG*CI-WTCOST
      GRTH=GRTH+GI
      GO TO 167
C  CALCULATION OF THE COST OF IRRIGATION
```

```
 60 FNR=0.-SSIG*CI-SWTCOS
    HSMA=.001
161 FNR=FNR
    GSUM=0.
    GR=0.
    GRTH=0.
    DEAD=1.
C  SUMMATION OF ALL VARIABLES OVER THE WHOLE GROWING SEASON
167 SUMEV=SUMEV+EV(K)
    SUMETT=SUMETT+ETT
    SUMEAT=SUMEAT+EAT
    SUMRF=SUMRF+RF(K)
    SEAAIG=SEAAIG+EAT
    SRFAIG=SRFAIG+RF(K)
144 CONTINUE
944 PRINT 45,K,EV(K),F(K),ETT,PT,EAT,RF(K),RBT,SMA,WTN,CAPA,
   1GR
C  STARTING THE IRRIGATION SEASON (45 DAY AFTER PLANTING)
945 IF(K-45)304,305,304
305 TSM=70.
    SEAAIG=0.
    SRFAIG=0.
C  ENDING THE IRRIGATION SEASON
304 IF(K-135)4,100,105
100 PRINT 103,FNR,GRTH,SIG
    PRINT 168,SUMEV,SUMETT,SUMEAT,SUMRF,SEAAIG,SRFAIG
    DEAD=1.
105 IF(K-150)4,303,303
 48 CALL EXIT
    STOP
    END
```

APPENDIX C. COMPUTER PROGRAM FOR DYNAMIC PROGRAMMING MODEL

```
C THIS PROGRAM HAS BEEN ADAPTED AND MODIFIED FROM THE ORIGINAL
C  CODING OF DUDLEY (1969)
C
C VARIABLES USED:
C  MOST OF THE VARIALES USED ARE THE SAME AS IN THE SIMULATION
C  MODEL GIVEN IN APPENDIX B.
C BWS           BEGINNING-STAGE WATER SUPPLY,IN.
C BSM           BEGINNING-STAGE SOIL MOISTURE LEVEL,IN.
C TSM           TERMINAL SOIL MOISTURE LEVEL(%)
C Z             FACTOR TO CONVERT FREQUENCIES OF EVENTS OCCURING
C               INTO PROBABILITIES
C FAV           MEAN OF THE VALUES OF RETURN CORRESPONDING TO
C               THE TRANSITION MOISTURE FROM ONE CLASS INTERVAL
C               TO ANOTHER DURING A STAGE$($)
C FKWA          PROBABILITY OF THE ENDING-STAGE SOIL MOISTURE
C               FALLING WITHIN A GIVEN CLASS INTERVAL
C FNETR         EXPECTED RETURN FROM MAINTAINING AN OPTIMAL
C               POLCY OVER THE REMAINDER OF SEASON ($)
C SUMT          EXPECTED RETURN OVER THE JUST-ENDED STAGE AND
C               ALL REMAINING STAGES,FROM MAINTAINING A GIVEN
C               POLCY IN THE JUST-ENDED STAGE AND OPTIMAL
C               POLICIES OVER ALL REMAINING STAGES ($)
C XAV           EXPECTED RETURN OVER THE CURRENT STAGE FROM
C              MAINTAINING A GIVEN POLCY IN THE JUST-ENDED
C               STAGE ($)
C SMAK          PROBABILITY OF THE ENDING STAGE SOIL MOISTURE
C               FALLING WITHIN A GIVEN CLASS INTERVAL
C SMBK          SUM OF THE VALUES OF RETURN CORRESPONDING TO
C               THE TRANSITION OF SOIL MOISTURE FROM ONE CLASS
C               INTERVAL TO ANOTHER DURING A STAGE ($)
C  DEFINE THE FORMAT AND DIMENSION STATEMENT
      DIMENSION FKWA(9),SMAK(6),SMBK(6),FAV(6),STABLE(9,6),
     1TABLE(9,6),HTABLE(9,6)
      DIMENSION EV(15),RF(15),TT(10,2),F(15),VM(15),RFS(20,
     115),EVS(20,15),WT(15)
    1 FORMAT(9X,F5.2)
  471 FORMAT(15F4.3)
  600 FORMAT(15F4.2)
  559 FORMAT(4F8.3)
  557 FORMAT(15F5.2)
  558 FORMAT(15F5.2)
  763 FORMAT(6F7.2)
  761 FORMAT(7F7.2)
C  INITIALIZE THE STAGE COUNTER AND BWS CLASS INTERVAL COUNTER
C  AND READ DATA WHICH APPLIES TO ALL STAGES
      NSTAGE=0
  555 READ(5,1)TT
 4849 NSTAGE=NSTAGE+1
      I9=0
```

```
C  READ AND MODIFY DATA WHICH APPLIES ONLY THE CURRENT STAGE
C  AND INITIALIZE SOME VARIABLES
       READ(5,557)F
       READ(5,558)VM
       READ(5,558)WT
       READ(5,559)CI,BK,SCAPA,COSTWT
       NYEAR=20
       Z=.05
       LS=1
       LDIR=0
       DO 9472 J=1,NYEAR
       READ(5,600)RF
       READ(5,471)EV
       DO 9482 K=1,15
       EV(K)=EV(K)
 9485  IF(RF(K)-.10)9481,9473,9473
 9481  RF(K)=0.
 9473  RF(K)=RF(K)
       RFS(J,K)=RF(K)
       EVS(J,K)=EV(K)
 9482  CONTINUE
 9472  CONTINUE
C  INITIALIZE AND INCREAMENT BWS,INCREMENT THE BWS CLASS
C  INTERVAL COUNTER AND INITIALIZE THE SM COUNTER
       BWS=-3
  604  BWS=BWS+3.
       I9=I9+1
       I6=0
       IF(BWS-24.)605,605,4848
C  INITIALIZE AND INCREMENT BSM WHEN RELEVANT, AND STORE THE
C  MAXIMUM EXPECTED RETURN AND OPTIMAL TSM POLICY WHEN ALL
C  TSM POLICIES HAVE BEEN CONSIDERED FOR THE CURRENT SPECIFIC
C  BSM LEVEL
  605  BSM=-30.
  602  BSM=BSM+20.
       IF(BSM)740,740,741
  740  SSMAXH=0.
       TTSM=0.
       GO TO 9613
  741  IF(LDIR)48,9603,9613
 9613  I6=I6+1
       STABLE(I9,I6)=SSMAXH
       HTABLE(I9,I6)=TTSM
       IF(BSM)788,788,9603
  788  BSM=BSM+20.
 9603  IF(BSM-90.)603,603,604
C  INITIALIZE AND INCREMENT TSM WHEN RELEVANT
  603  TSM=0.
       LDIR=1
```

```
      GO TO 9876
 9875 TSM=-10.
      LDIR=2
 9874 TSM=TSM+20.
      IF(TSM-90.)9876,9881,602
 9881 TSM=80.
C  SET CLASS INTERVAL CONTENTS AND ASSOCIATED VARIABLES TO
C  ZERO
 9876 DO 483 J=1,9
      IF(J-6)484,484,483
  484 SMAK(J)=0.
      SMBK(J)=0.
      FAV(J)=0.
  483 FKWA(J)=0.
      IF(TSM)48,1234,1235
C INITIALIZE THE 'MAXIMUM RETURN' TO A LARGE NEGATIVE VALUE
C  SO THAT THE INITIAL VALUE WILL NEVER BE THE GREATEST VALUE,
C  EVEN IF ALL LATER VALUES WERE NEGATIVE BECAUSE OF CROP DEATH
 1234 SSMAXH=-1000.
      TTSM=0.
 1235 LS=1
      HGIS=0.
      KYEAR=0
  450 DO 1944 K=1,15
      RF(K)=RFS(LS,K)
      EV(K)=EVS(LS,K)
 1944 CONTINUE
C  INITIALIZE VARIABLES SIMILAR TO SIMULATION MODEL GIVEN
C  IN APPENDIX B.
      LS=LS+1
      CAPAMX=12.15
      GRAVDY=1.
      KYEAR=KYEAR+1
      SM=BSM
      WS=BWS
      HSMA=SM
      SMA=SM
      RBT=0.
      SIG=0.
      WTCOST=0.
      GSUM=0.
      SMB=0.
      CAPA=SCAPA
      GISUM=0.
      WTNG=0.
      GRTH=0.
      SSIG=0.
      GRAVWT=0.
      DAY=GRAVDY+1.
```

```
      RUNOFF=0.
      WB=0.
      K=0
    4 K=K+1
C   START SIMULATING THE CROP GROWTH FOR EACH 6 STAGES
      DAY=DAY+1.
      PT=0.
      WTN=0.
      ETT=F(K)*EV(K)
  771 RBTMX=1.59-.0159*HSMA
  772 IF(RF(K)+RBT-RBTMX)721,721,720
  720 RBT=RBTMX-ETT
      GO TO 207
  721 RBT=RBT+RF(K)-ETT
  207 IF(RF(K))92,92,98
   92 IF(RBT)97,97,98
   97 RBT=0.
      GO TO 220
   98 PT=1.
  697 IF(RBT)93,93,220
   93 RBT=0.
  220 IF(CAPAMX-CAPA)221,221,209
  209 HSMA=(((CAPA*HSMA*.01)+.17357)/(CAPA+.17357))*100.
      CAPA=CAPA+.17357
  221 IF(PT)7,7,26
    7 AE=HSMA/10.
      KAE=AE
      FKAE=KAE
      K1=KAE+1
      IF(FKAE-AE)25,10,25
   10 K1=K1-1
   25 K1=10-K1+1
      IF(K1-10)8,8,9
    9 K1=10
      IF(K1.LE.0)K1=1
    8 K2=1
      IF(EV(K).GT..30)K2=2
      PT=TT(K1,K2)
   26 EAT=PT*ETT
  210 IF(HSMA+(RF(K)*100./CAPA)-(EAT*100./CAPA)-TSM)58,58,216
   58 WTN=(100.-HSMA)*CAPA/100.
      IF(WS-WTN)217,59,59
   59 IF(SIG)725,780,725
  780 COSWT=0.
      GO TO 598
  725 COSWT=COSTWT
  598 WS=WS-WTN
      SIG=SIG+WTN
      WTCOST=WTCOST+COSWT
```

```
      WTNG=WTNG+1.
      SMA=((HSMA*CAPA*.01)+WTN+RF(K))*100./CAPA
      GO TO 41
  217 WTN=0.
  216 SMA=((HSMA*CAPA*.01)+RF(K)-EAT-RUNOFF)*100./CAPA
   41 IF(SMA-99.9)51,218,218
  218 SMA=100.
      DAY=0.
      RBT=0.
   51 IF(DAY-GRAVDY)699,699,698
  699 SMA=100.
  698 HSMA=SMA
   50 IF(SMA)60,60,61
   61 IF(PT-1.)54,62,55
   54 GI=1.-WT(K)
      GO TO 63
   62 GI=1.
      GO TO 63
   55 GI=0.
   63 GR=GI*VM(K)
      GO TO 144
   60 FNR=BK-SIG*CI
      GO TO 106
  144 GSUM=GSUM+GR
      GISUM=GISUM+GI
      IF(K-15)4,488,488
C  CALCULATION OF THE RETURN FOR EACH STAGE
  488 FNR=GSUM-SIG*CI-WTCOST
      HGIS=HGIS+GISUM
  106 SM=SMA
      KWS=WS
      KWS=KWS+2.5
      KSB=KWS/5+1
      KWS=5*(KSB-1)
      IF(KWS-40)100,101,101
  100 FKWA(KSB)=FKWA(KSB)+1.
      GO TO 102
  101 FKWA(9)=FKWA(9)+1.
      KWS=40
  102 IF(SM)104,104,105
  104 SM=0.
      KSB=1
      GO TO 110
  105 IF(SM-80.)108,107,107
  107 SM=90.
      KSB=6
      GO TO 110
  108 KSM=SM
      KSM=KSM/20
```

```
      KSB=KSM+2
      SM=KSM*20+10
  110 SMAK(KSB)=SMAK(KSB)+1.
      SMBK(KSB)=SMBK(KSB)+FNR
      YEARN=NYEAR
      IF(KYEAR-NYEAR)450,460,48
  460 AVGDYS=HGIS/YEARN
C  CONVERT THE NUMBER OF OBSERVATION OF WS IN EACH CLASS
C  INTERVAL TO PROBABILITIES OF OCCURANCE OF SUCH WS LEVELS
  733 DO 463 J=1,9
  463 FKWA(J)=FKWA(J)*Z
      DO 464 J=1,6
      IF(SMAK(J))48,700,701
  700 FAV(J)=0.
      GO TO 464
  701 FAV(J)=SMBK(J)/SMAK(J)
C  CONVERT THE NUMBER OF OBSERVATION OF SM IN EACH CLASS
C  INTERVAL TO PROBABILITIES OF OCCURANCE OF SUCH SM LEVELS
  464 SMAK(J)=SMAK(J)*Z
      XAV=0.
      DO 8465 J=1,6
 8465 XAV=XAV+FAV(J)*SMAK(J)
C  STSRT CALCULATING THE EXPECTED RETURN FOR ALL COMBINATION
C  OF BWS, BSM AND TSM
  731 IF(NSTAGE-1)734,734,735
  734 IF(XAV-SSMAXH)1236,1236,1237
 1237 SSMAXH=XAV
      TTSM=TSM
      GO TO 1236
  735 SUMT=0.
      DO 9865 I=1,9
      IF(FKWA(I))9865,9865,9866
 9866 DO 9867 J=1,6
      IF(SMAK(J))9867,9867,9868
 9868 PTAV=(TABLE(I,J)+FAV(J))*SMAK(J)*FKWA(I)
      SUMT=SUMT+PTAV
 9867 CONTINUE
 9865 CONTINUE
      IF(SUMT-SSMAXH)1236,1236,1268
 1268 SSMAXH=SLMT
      TTSM=TSM
 1236 GO TO(9875,9874),LDIR
 4848 DO 760 I=1,6
      DO 760 J=1,9
  760 TABLE(J,I)=STABLE(J,I)
      DO 764 J=1,9
      WRITE(6,763)(HTABLE(J,I),TABLE(J,I),I=1,6)
  764 CONTINUE
      IF(NSTAGE-6)4849,48,48
   48 STOP
      END
```